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THE

PRIMER OF HYDRAULICS

BY

FREDERICK A. SMITH

Hydraulic Engineer

CHICAGO, ILLINOIS, U. S. A.

CHICAGO:
D. H. ANDERSON
PUBLISHER

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PREFACE

While designing the Intercepting Sewer System of the city of Chicago during the period 1897 to 1907, the author was frequently compelled to spend considerable time in the preparation of tables and diagrams to enable him to analyze the various problems entering into the general scheme. Thus he discovered that there was a scarcity of hydraulic information in regard to the flow of water in circular conduits flowing partially full, which fact induced him to compute a number of tables covering all such cases in applied hydraulics. These tables have since been supplemented and perfected and have been embodied into this work together with much other new matter so that the "Primer of Hydraulics" in its present condition contains sufficient information to solve any problems relating to the flow of water in any channels utilized in municipal, drainage or irrigation work or any other problems arising from the flow of water in channels.

The tables, giving the factor *C* for all practical sizes of hydraulic radii either directly or by interpolation, will be found especially valuable, and the many practical problems worked out in full should prove highly important as a guide in the use of the book to those who have not had the advantage of a higher engineering education.

The matter has been grouped progressively, illustrating the simpler principles first, and advancing to the more complicated ones, thus providing a handbook possessing considerable value from an educational point of view. In fact, the author has designed the work on lines enabling anyone with a grammar school education to grasp the subject and by following the course carefully to become proficient in the analysis of ordinary hydraulic problems.

The hydraulic engineer will find much in this work which will be of great value to him in the use of the tables and methods of analysis contained therein.

The author does not claim, by any means, that the Primer of Hydraulics, in its present form, is a perfect work, for it is not, and will appreciate the favor if readers and users of the book tell him freely their opinion of the same, and especially call his attention to errors contained therein, so that future editions may be made more perfect and contribute to the general advance in the application of the science of hydraulics to the solution of the many practical problems which the world is now facing everywhere; problems relating to the improvement in sanitation, drainage, irrigation, ship canals and municipal water supplies.

THE PRIMER OF HYDRAULICS.

INTRODUCTION.

Everything round about us which makes its presence known to us by acting upon our senses is called *Nature*. Thus nature is the sum total of everything capable of making an impression upon the human senses. It may be divided into two large subdivisions, namely, *Things* and *Phenomena*, and the knowledge of them is called *Natural History* and *Natural Science* respectively; thus the knowledge of animals, vegetation and minerals is called Natural History, subdivided into *Zoology, Botany* and *Mineralogy*. The knowledge of phenomena, such as the flowing of water in a river, or the freezing of water, or the rusting of iron and the growing of vegetation, *is* called *Natural Science*. The many thousands of phenomena in nature can be divided into two great classes. The one class embraces phenomena which do not change the constitution of the things upon which they are observed; for instance, the sounding of a bell, the breaking of a glass, the falling of an apple, the melting of wax, the boiling of water, are all phenomena which do not change the character of the substances involved; thus the bell is still a bell after sounding; the broken glass is still glass, the apple is still an apple after its fall, the wax is still wax after it is melted and the steam is still water after condensation. Such phenomena are called *Physical Phenomena* and the science teaching their knowledge is called *Physics*.

The other class of phenomena always produces a change in the affected bodies; for instance, if a copper coin is immersed into nitric acid there will be observed an intense action, the fluid turning green with a rising of brown vapor, and if enough nitric acid is used the entire copper coin disappears; if the green liquid is boiled down crystals of a green substance appear, which are neither copper nor nitric acid, but is a new substance called copper nitrate; thus the phenomenon has changed the constitution of the copper as well as that of the nitric acid; such phenomena are called chemical phenomena and the science treating of them is called *Chemistry*. Some other chemical phenomena are, for instance, the rusting of iron, the burning of wood, and the various processes of life in animals and plants, the manufacture of glass, reduction of ores, etc.

From the foregoing it is apparent that the phenomena of Hydraulics, upon which this work will treat principally, belong into the field of Physics, as the action of the forces which produce pressure and flow in water and other fluids does not produce a chemical change in its constitution. As this book is intended to treat the subject of Hydraulics in a practical way, adapted to the needs and capacities of the practical men who follow the various application of Hydraulics, the author will devote some time to the most important elementary principles underlying this subject, as an understanding of them is necessary for the comprehension of Hydraulics and its application to practical problems.

ARTICLE I.

GENERAL PROPERTIES OF MATTER.

Everything which possesses weight is called *matter.* Even the lightest substance known, Hydrogen Gas, has weight, but one cubic inch of Hydrogen Gas weighs only the 11,000th part of a cubic inch of water, and a cubic inch of air is nearly 14½ times as heavy as a like quantity of Hydrogen Gas.

Following are the general properties of matter:

1. *Extension.* This means that all matter requires space for its existence. This is self-evident.

2. *Impenetrability.* This means that two bodies cannot occupy the same space at the same time.

3. *Divisibility.* This means that all matter can be divided into smaller parts; the truth of this is also self-evident. Thus if we crush a piece of chalk until it is a fine powder and look at it through a strong microscope, we observe that the apparently fine powder is composed of lumps of different forms, and though we are practically unable to break them into smaller parts, we can readily imagine that such division can be carried on much further. By the same process of reasoning a limit of divisibility must, however, be reached some time, or, with other words, there must be a smallest particle of every kind of matter. Such smallest particle is called a *Molecule,*

Some kinds of substances are composed all through of just one kind of matter; for instance, the diamond, which is pure carbon, while other substances are composed of two or more kinds of matter, as, for instance, water. Bodies composed of but one kind of matter are called *Elements,* while such bodies as are composed or two or more elements are called *Compounds.* If the divisibility in an element is carried on in imagination until the smallest particle is reached, then we are face to face with the *Atom;* hence an atom can be defined as the smallest particle of matter which can be divided no further. On the other hand, the smallest particle of a compound, the molecule, can be divided into the elementary atoms composing it. For instance: Water is composed of two elements, Oxygen and Hydrogen, and it may be decomposed by an electric current into these two elements; when this is done the Hydrogen gas appears on one electrode and the Oxygen on the other; also the volume of the Hydrogen gas which appears in tube B, Fig. 1, is equal to twice the volume of the Oxygen rising in tube A; this shows that for every atom of Oxygen there are two atoms of Hydrogen in a molecule of water. The elementary atom is represented by a symbol, the letter "O" in chemical lore represents an atom of Oxygen the letter H represents an atom of Hydrogen, so that the chemical symbol for water is H_2O, which means that the molecule of water consists of two atoms of Hydrogen and one atom of Oxygen. This symbol also tells the weight relation between the two elements, which is done by a table of atomic weights in which the relative weight of the various elements are compared with that of Hydrogen, whose weight, being the lightest substance known, is taken as unity; thus the atomic weight of Oxygen is 16, which means that the Oxygen atom is 16 times heavier than Hydrogen. This then gives the information that if, for instance, 9 pounds of water are decomposed into Oxygen and Hydrogen, we obtain 8 pounds of Oxygen and one pound of Hydrogen.

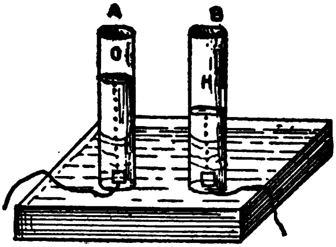

Fig. I.

Water has purposely been chosen as an example, as this work will deal principally with water and its properties, and therefore the reader should increase his knowledge constantly in this direction.

4. *Porosity.* This means that the atoms or molecules

composing any body do not touch each other, but are separated from each other by vacant spaces. That this is so is plain, from the fact that the size of any body will change under certain conditions without changing the amount of matter in the body. Thus in Fig. 2 in the square A B C D is shown an aggregation of 16 atoms, having spaces between them representing the pores, which is the property of porosity.

5. *Expansibility and Contraction.* These two properties mean that the volume of bodies under the influence of certain forces may increase or diminish. The forces producing such changes are variations in temperature and pressure. Thus if a bar of iron is heated it expands in length as well as in width, and if in Fig. 2 A B C D is considered a square plate of steel which under the influence of heat is expanded to the size E, F, G, H, it is plain that as the weight remains the same, the number of atoms has not been increased and it is the pores which have become larger or, with other words, the particles have moved away from each other. This same effect may be produced by applying external force tending to pull the molecules or atoms away from each other.

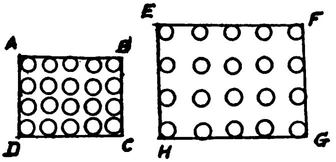

Fig. 2.

The reverse phenomenon or contraction takes place when a drop in temperature occurs, or when the body is subjected to pressure, tending to force the particles together. This is seen in Fig. 2 when considering the atoms in E F G H, moving closer together when the volume decreases, as indicated in A B C D.

6. *Indestructibility.* This general property of all bodies means that no matter can be either destroyed or created, and that the total sum of atoms in nature remains unchanged. That this is so is perhaps not always easily seen, but a little study and thought will always confirm this statement. For in-

stance, if a ton of coal is burned the residual ashes may weigh only a few hundred pounds; what has become of the remaining matter? Chemistry proves that the remainder has been converted into volatile substances which have passed off in the form of smoke and vapor, and that there has nothing been lost; the hydrogen contained in the coal has been combined with the oxygen of the air to form water, and the carbon has united with other portions of the oxygen of the air to form carbon dioxide, both combustion products being in the form of an invisible vapor have passed into the outer air.

But there is no particle of matter ever lost in the economy of nature, and the different elements merely change position when going through chemical or other changes.

7. *Inertia.*—This means that all bodies have the tendency to remain in their state of rest or state of motion, and that it takes some external force to change such state. Thus, a wagon standing still will remain standing still until some outside force makes it move; this principle is easily seen, but the other part of the claim that bodies in motion will remain in motion until stopped by some external force requires some study to prove in all cases; for instance, roll a ball along the ground and it is observed that its motion grows slower and slower and finally comes to rest. This looks at first to be a violation of the principle of Inertia, yet it is in full accord therewith, for the stoppage of the motion of the ball is caused by two external causes or forces, namely, the impenetrability of the air which offers a certain resistance to the rolling ball, and the friction upon the surface on which the ball rolls bring it gradually to a stop.

The phenomena of motion and the causes producing them will be further treated and in more detail in the article on Mechanical Principles.

ARTICLE II.

ALGEBRAIC PRINCIPLES.

A knowledge of mathematical principles is necessary for the successful application of scientific principles to practical problems. This applies also to hydraulics and in order to convey this knowledge to those who have not had it before, or to refresh it in those who have forgotten it articles 2 and 3 have here been introduced.

Algebra is a general arithmetic teaching the principles of arithmetic by the use of symbols. It is presumed that the four elementary operations, addition, subtraction, multiplication and division are known so far as common numbers are concerned and the following explanations illustrate their adaptability to algebraic symbols:

1. *Addition.*—The act of adding two or more numbers together means to find a new number which contains as many units as the two or more given numbers; thus: $7+3=10$ (read: 7 plus 3 equal 10) represents that operation arithmetically; the algebraic representation is just like it: $a+b=c$ (read: a plus b equal c) and it means that the number of units in a are added to the number of units in b and that the number c contains as many units as there are in a and b together. The numbers to be added together are called ***summands*** and the result of the addition is called the ***sum.*** Thus a and b are summands and c is the sum.

The expression $a+b=c$ is also called an equation, be cause the value of $a+b$ is equal to c; $a+b$ is the left side and c the right side of the equation and as the principle of equation is very important in this work the following axioms should be mastered right now: If two things are equal they remain equal if the same quantity is added to each, or if the same quantity is subtracted from each, or if each is multiplied by the same number, or if each is divided by the same number. That these principles are true is easily seen. Thus, if

$7+3=10$ then

$7+3+2=10+2$ (adding 2 to both sides),
$7+3-5=10-5$ (subtracting 5 from both sides),
$7+3\times4=10\times4$ (multiply both sides by 4) and
$\frac{7+3}{2}=\frac{10}{2}$ dividing both sides by 2; for in the first operation we have $12=12$; in the second operation $5=5$; in the third operation $40=40$, and in the fifth operation we have $5=5$. The application of these principles applies with equal force to algebraic equations as will be seen hereafter.

2. *Subtraction.*—Unknown quantities are usually expressed by the symbol x, y, z, etc., though any letter may be used for that purpose. If in the expression:

$$7+x=10$$

one of the summands x is unknown, it is found by subtracting the other summand 7 from both sides, thus:

$$7+x-7=10-7$$

The equality of the two sides has not been disturbed by subtracting 7 units from both sides; on the left side this leaves x and on the right side it leaves $10-7$, hence:

$$x=10-7$$
$$x=3$$

This is a very important principle and the expression $10-7=3$ shows how a summand is found by subtracting the given summand from the sum. It also illustrates the principle of subtraction. The number 10 from which is subtracted is called the ***minuend,*** the number 7 which is subtracted is called the ***subtrahend,*** and the number 3 which is the result of the subtraction is called the ***difference.***

As a general thing when in an equation all but one quantity are given then that one unknown quantity can be found. Thus, if in the above equation the number 10 was not known we give it the symbol x and then write $x-7=3$. Now, in order to find x, add 7 to both sides of the equation:

$$x-7+7=3+7$$

On the left side $-7+7$ cancel, hence:

$$x=3+7$$

This also shows the principle that the minuend is equal to the subtrahend plus the difference.

3. *Negative Numbers.*—In common arithmetic all numbers are greater than zero; in algebra, however, we say there are just as many numbers smaller than zero than greater than zero—or with other words there is an ascending series of numbers greater than zero and a descending series of numbers smaller than zero; the former are called positive numbers the latter negative numbers. The positive numbers are indicated by the + (plus) sign though position numbers need not have the sign affixed, thus the numbers 7 or A indicate position numbers while —7 or —A would mean negative numbers. A negative number bears the same relation to a position number as debts bear to property, thus adding a negative number to a positive number diminishes the positive number as many units as there are in the negative number. Thus: adding 4 dollars debts to 7 dollars property would leave 3 dollars property. This would be indicated thus:

$$+7+(-3)=+4$$

On the other hand to subtract a negative number from a positive number increases the latter as many units as the negative number has, thus:

$$+7-(-3)=+10$$

Also if a negative number is added to a negative number it decreases the value of that negative number, thus:

$$-7+(-3)=-10$$

And if a negative number is subtracted from a negative number it increases the value of that negative number:

$$-7-(-3)=-4$$

The value of the number—4 is greater by 3 units than the number —7. Also the sum of a positive and negative number of equal units is equal to zero, thus:

$$+7-7=0$$

4. *Multiplication.*—In a sum like: $5+5+5$ where the summand 5 appears three times the operation is indicated by writing the equal summand but once, and the number which

shows how often that equal summand is present is written along with the equal summand with the symbol of multiplication, thus: 5×3 or 3×5 and the result being 15, we have:

$$3\times5=15$$

This operation is called multiplication; the numbers 3 and 5 are called factors and the number 15 is called the product.

In a similar manner if we have the sum: $a+a+a+a$ we could say $4\times a$ or $4a$; thus, when numbers are expressed by letters no multiplication sign need to be placed between, hence, $7x$ means seven times x, ab means a times b, abc means a times b times c, etc. When a sum or a difference is to be multiplied by a number, each term of the sum and difference must be multiplied, and the sum or difference placed in brackets, for instance: Multiply the sum $4+5$ by 7; this should be written thus: $7\ (4+5)$; this would be equal to $7\times4+7\times5$ or $28+35$ which is equal to 63; this is obvious from the fact that $4+5=9$ and $7\times9=63$; hence, if instead of given numbers we have algebraic symbols the same principles obtain, thus:

$$a\ (b+c)=ab+ac$$

Likewise: $a\ (b-c)=ab-ac$

This principle also governs if sums and differences are multiplied by sums and differences, for example: Multiply $9-2$ by $4+7$; $(9-2)\times(4+7)$ multiply each term of one with each term of the other. thus:

$$9\times4+9\times7-2\times4-2\times7=$$
$$36+63-8-14=$$
$$99-22=77$$

That this is correct is seen if, instead of $9-2$ we write 7, and instead of $4+7$ we write 11, then $7\times11=77$.

Example: Multiply $11-5$ with $8-3$; this should be written: $(11-5)\ (8-3)$ and the result should be 30, since $11-5=6$ and $8-3=5$; multiply as above:

$$11\times8-11\times3-5\times8+5\times3=88-33-40+15=30$$

Observe that the product of two positive or two negative numbers is positive and that the product of a positive with a negative number is negative. These principles are necessary to understand when working with algebraic terms or formulas.

Thus, multiply: $(a+b)$ with $(c+d)=(a+b)\ (c+d)=ac+ad+bc+bd$.

Also $(a+b)\ (c-d)=ac-ad+bc-bd$; also $(a-b)\ (c-d)=ac-ad-bc+bd$.

When expressions occur in which like numbers appear after multiplication they are united thus:

$$(a+b)\ (a+b)=aa+ab+ab+bb$$

Where the same number is multiplied by itself, like aa and bb, it is called a square and is written a^2 or b^2; also as the number ab appears twice instead of $ab+ab$ we say $2ab$, thus instead of $aa+ab+ab+bb$ write $a^2+2ab+b^2$.

This expression also illustrates an important principle that the square of the sum of two numbers is equal to the sum of the squares of the two numbers plus twice their product. To illustrate an application of this multiply 203 by 203.

Solution: Instead of 203 set $(200+3)\ (200+3)$ this is equal according to the above principle:

$$200\times200+3\times3+2\times3\times200=40000+9+1200=41209$$

Another principle is found by multiplying a difference by itself, thus: $(a-b)\ (a-b)=a^2-ab-ab+b^2=a^2-2ab+b^2$. This principle says: The square of the difference of two numbers is equal to the sum of the squares of these two numbers minus twice their product. Application, multiply 98 by 98.

Solution: Set $100-2$ for 98, then we have: $(100-2)\times(100-2)=100\times100-2\times100-2\times100+4=10000+4-400=9604$.

Another principle is illustrated as follows:

Multiply the sum of two numbers by their difference, as $(a+b)\ (a-b)=a^2+ab-ab-b^2$.

In this answer $+ab$ and $-ab$ cancel, hence the answer is a^2-b^2. This principle, expressed in words, says: The product of the sum of two numbers into their difference is equal to the difference of their squares. This is also true in the reverse way, for instance: Subtract the square of 13 from the square of 17, which operation would ordinarily require, first, to multiply 17 by 17, next multiply 13 by 13, and then subtract the latter product from the former, thus: $17\times17=289$, $13\times13=169$; $289-169=120$. By the aid of the above principle this problem is solved much easier as the answer will be found by multiplying the sum by the difference, thus: $17^2-13^2=(17+13)\ (17-13)=30\times4=120$.

5. *Division.*—If in a product one factor is unknown it can be found when all the other quantities are known, for instance: $3x=21$, which means to find a number which, multiplied by 3, gives 21; this number is evidently 7, because 3 times 7 equal 21, and is found by dividing 3 into 21; hence, the operation of finding a factor is called division; the operation is indicated thus:

$$x=\frac{21}{3} \text{ or } x=21\div3$$

In this expression the number 21 is called the dividend or numerator, the number 3 is called the divisor or denominator and the number x (or 7) is called the quotient or fraction. As $\frac{21}{3}=7$ the relation between the different terms may be expressed that the dividend is equal to the quotient multiplied by the divisor, and that the divisor is equal to the dividend divided by the quotient.

The rules for the signs in division are the same as in multiplication; thus, like signs produce positive, and unlike signs produce negative numbers, for instance:

$$a\div b=\frac{a}{b};\ a\div -b=-\frac{a}{b};\ -a\div b=-\frac{a}{b};\ \text{and } -a\div -b=\frac{a}{b}$$

This general law regarding the signs may be shortly expressed thus: Like signs produce positive and unlike signs produce negative results.

6. *Involution.*—When in a product all the factors are equal for instance, $5\times5\times5\times5$, then, instead of writing the factor 5 four times, we write it but once, and the number 4 which indicates how many times said factor is to be multiplied by

itself on the upper right hand of the number 5, making the number 4 about half the size of the 5, thus:

$$5^4=625$$

Which is read: 5 to the 4th power equals 625. The number 5 is called the base, 4 is called the exponent and 625 is called the power. Thus, the power is found by multiplying the base as often with itself as the exponent indicates. Thus, 2^5 means that the number 2 is multiplied 5 times by itself, or $2\times2\times2\times2\times2=32$.

Also *aaaaaa* should be written a^6, $xxxxxxx=x^7$. If two powers of the same base are divided into each other, the resulting quotient can be simplified, for instance:

$$\frac{3^5}{3^2} = \frac{3\times3\times3\times3\times3}{3\times3} = \frac{3\times3\times3}{1} = 3^3 = 3^{5-3}$$

$$\frac{3^5}{3^3} = \frac{3\times3\times3\times3\times3}{3\times3\times3} = \frac{3\times3}{1} = 3^2 = 3^{5-3}$$

$$\frac{3^5}{3^4} = \frac{3\times3\times3\times3\times3}{3\times3\times3\times3} = \frac{3}{1} = 3^1 = 3^{5-4}$$

$$\frac{3^5}{3^5} = \frac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3} = 1 = 3^{5-5} = 3^0$$

$$\frac{3^5}{3^6} = \frac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3\times3} = \frac{1}{3} = \frac{1}{3^1} = 3^{5-6} = 3^{-1}$$

$$\frac{3^5}{3^7} = \frac{1}{3^2} = 3^{5-7} = 3^{-2}$$

From the above it appears that powers of the same base are divided into each other by subtracting the exponent of the divisor from the exponent of the dividend. Also, that a power having the exponent zero equals 1; also, that a power with a negative exponent represents a fraction with the numerator 1 and the denominator is the power with positive exponent. Illustrating the same principles upon algebraic terms we have:

$$\frac{a^{13}}{a^{11}} = a^{13-11} = a^2$$

$$\frac{a^{13}}{a^{13}} \quad a^{13-13} = a^0 = 1$$

$$\frac{a^{13}}{a^{17}} = a^{13-17} = a^{-4} = \frac{1}{a^4}$$

If two powers of the same base are multiplied together it is done by adding the exponents together, thus: $a^3 \times a^4 = a^7$, $(b+c)^5 \times (b+c)^8 = (b+c)^{13}$.

If, however, a power is to be raised to another power like x^5 is to raised to the third power then it is indicated thus: $(x^5)^3 = x^{15}$, which means that the exponents must be multiplied.

NOTE: The second power of a number is called the square, and the third power is called the cube of such number.

7. *Evolution.*—In the expression: $4^3 = 64$ are three quantities, the base 4, the exponent 3 and the power 64, and if any two of them are given, the third one may be found. Involution showed how the power is found when the base and the exponent are given. Thus, the power 64 in the above term is found by multiplying the base 4 three times by itself. Suppose, however, the power 64 and exponent 3 are given and it is required to find the base 4, then the problem is to split the power 64 into as many equal factors as the exponent 3 indicates. This operation is called evolution and is indicated thus:

$$x = \sqrt[3]{64}$$

Read: x=the third root of 64; x is now called the root, 3 is the index, 64 is the radical and the sign between 3 and 64 is called the radical sign. If the index is 2 it is called the square root, and is usually omitted; if the index is 3 it is called the cube root; if the index is 5 it is called the fifth root, etc. In solving the various problems in applied hydraulics it is occasionally necessary to extract the square root, cube root, fifth and seventh root. The extraction of the square root is a familiar operation with many people, but the cube root is much more difficult, and higher roots are usually extracted by the use of logarithms. Tables of square roots and cube roots are appended, especially selected for the use in hydraulic work and the following paragraph gives hints, how to find any root by the use of logarithms.

8. *Logarithms.*—In 6 it was stated that powers of the same base are multiplied together by adding the exponents, that

powers of the same base are divided into each other by subtracting the exponents, that powers are raised to another power by multiplying the exponent by the other power; inversely the root may be extracted from any power by dividing the exponent of the power by the index of the root. To illustrate these principles briefly, we will use the base 10; then:

$$10^2 \times 10^3 = 10^{2+3} = 10^5$$

$$10^6 \div 10^2 = 10^{6-2} = 10^4$$

$$(10^5)^3 = 10^{5\times 3} = 10^{15}$$

$$\sqrt[7]{10^2} = 10^{\frac{2}{7}}$$

Every number can be considered as a power of 10; thus:

$1 = 10^0$.
$10 = 10^1$.
$100 = 10^2$.
$1{,}000 = 10^3$.
$10{,}000 = 10^4$.
$100{,}000 = 10^5$.
$1{,}000{,}000 = 10^6$.

Also for number smaller than 1:

$.1 = 10^{-1}$.
$.01 = 10^{-2}$.
$.001 = 10^{-3}$.
$.0001 = 10^{-4}$.
$.00001 = 10^{-5}$.
$.000001 = 10^{-6}$.

These exponents of the base 10 producing the different numbers are called logarithms. Thus, it is seen the logarithm of 1=0, the logarithm of 10=1, of 100=2, of 1,000=3, of 10,000=4, etc. So the logarithm of all numbers between 1 and 10 are greater than 0 and smaller than 1; the logarithms of all numbers between 10 and 100 are greater than 1 and smaller than 2; the logarithms for all numbers between 100 and 1,000 are greater than 2 and smaller than 3; it shows also that all logarithms are decimal fractions, except the exact powers shown.

Tables of these logarithms have been computed and a table of them is embodied into this book.

9. *Formulas.*—A formula is an algebraic expression showing how to apply a scientific principle to the solution of a practical problem. They are always in the form of an equation and most of them are simple enough to be handled correctly by anyone having a common school education. As an illustration let the box

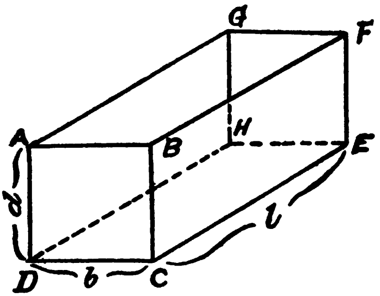

FIG. 3.

A, B, C, D, E, F, G, H, shown in Fig. 3 be of a rectangular shape; if the depth AD=3 feet, the width DC=4 feet, and the length CE=6 feet, then the volume enclosed by the box would be 3×4×6=72 cubic feet, and as a cubic foot of water weighs 62.5 pounds we could figure out quickly how much weight there would be in the box when filled with water.

If now the depth of the box is called d, the breadth b, the length l and the volume v, then the formula expressing the volume is as follow:

$$v=d\ b\ l$$

From this formula the depth d can be found by dividing both sides of the equation by bl:

$$d=\frac{v}{bl}$$

which teaches that the depth of box is found by dividing the volume by the product of breadth times the length.

Likewise, if it be required to find the breadth of a required box it would be done by dividing the volume by the product of length times depth, thus:

$$b=\frac{v}{ld}$$

and the length would be found by dividing the volume by the product of depth times breadth or:

$$l=\frac{v}{bd}$$

So the four formulas:

$$v=dbl,\ d=\frac{v}{bl},\ b=\frac{v}{dl},\ \text{and}\ l=\frac{v}{bd}$$

are all derived from the simple principles of equations which have been illustrated in this article.

Applied Problems. 1. Find the depth of a rectangular box if the width is 5 ft. the length 8 ft. and the volume equals 82 cubic ft. Solution: In formula: $d=\frac{v}{bl}$. Substitute the given quantities: $d=\frac{82}{5\text{x}8}=\frac{82}{40}=2.05$ ft.

2. Find the breadth of a box containing 100 cubic ft. if the depth is 2½ ft. and the length 16 ft.:

Solution: In formula: $b=\frac{v}{ld}$. Substitute the given quantities: $b=\frac{100}{16\text{x}2.5}=\frac{100}{40}=2.5$ ft.

3. Find the length of a box containing 1 gallon (231 cubic inches) if the depth is 4 in. and the breadth is 5 in. Solution: In formula $l=\frac{v}{bd}$ substitute the given quantities, then: $l=\frac{231}{5\text{x}4}=\frac{231}{20}=11.55$ inches.

This shows the usefulness of formulæ to solve practical problems.

ARTICLE III.

GEOMETRICAL PRINCIPLES.

1. Introductory. The science relating to the forms or shapes of things regardless of the substances enclosed is called *Geometry.* Elementary conceptions in geometry are *points, lines, surfaces and solids.* A point has no extention or size, but merely denotes location in space. A geometrical line has no breadth or thickness, but possesses *length.* A surface has no thickness, but has length and width, and a solid has length, breadth and thickness. All geometrical quantities are within *space,* which is everywhere and which is infinite. All geometrical figures drawn in a plane surface belong in the field of *plane geometry,* and all geometrical figures lying in more than one plane are part of *solid geometry.*

2. *Angles and Triangles.*

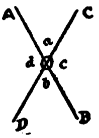

Fig. 4.

When two straight lines, AB and CD, intersect each other as shown in Fig. 4, they form *angles* with each other; let O be the point of intersection, then there are formed four angles. Angles are designated by naming three points like AOC, which angle includes the space bounded by AO and CO. The point O is called the vertex, and AO and CO are called the sides or legs of the angle; these sides must be considered of infinite length, so that the size of an angle is not affected by the length of its sides; sometimes angles are expressed also by writing a letter into the angle area near the vertex like a, x, α, β, etc. This simplifies the work many times; thus, if we set for angle AOC the letter a and for angle BOD the letter b, for angle BOC the letter c, and for angle AOD the letter d, then it is much easier to refer to any one of the angles. Angles in the position of a and b are called vertical angles, the legs of the one being extensions of the legs of the other; likewise c and d are vertical angles, and the first important principle in geometry is that vertical angles are equal; thus angle a = angle b and angle d = angle c. While angle a = angle b and angle c = angle d, it is seen that there is much difference between angles a and c and means are provided to compare different angles together by measuring them with a unit angle in the following manner:

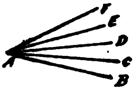

Fig. 5.

Conceive the whole plane divided into 360 equal angles, as indicated in Fig. 5, all radiating from the same point *A* like *BAC*, *CAD*, etc., then each of these angles is called 1 degree (1°). Each degree is again divided into 60 equal angles, called minutes ('), and each minute is divided into 60 equal angles called seconds ("); thus the whole plane measures 360°, or 21,600', or 1,296,000", so that by using these units any angle may be exactly expressed in degrees, minutes and seconds.

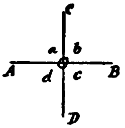

Fig. 6.

If two lines intersect each other so as to make four equal angles as in Fig. 6, in which the four angles formed are equal, $a = b = c = d$, then each angle equals $360 \div 4 = 90°$. Such angles are called right angles, and the lines forming them are said to stand perpendicular to each other; thus the line *AB* stands perpendicular to the line *CD* in the point *O*. An angle smaller than 90° is called an *acute* angle, like angle *ABC* in Fig. 7, and an angle greater then 90° is called an obtuse angle, like angle *DEF* in Fig. 8. If the sum of two angles equals 90°, like angles *a* and *b*, in Fig. 9, they are called *complementary* angles, and if the sum of two angles like *c* and *d*, in Fig. 10, is equal to 180°, they are called supplementary angles.

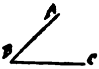

Fig. 7.

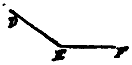

Fig. 8.

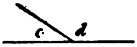

Fig. 10.

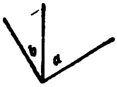

Fig. 9.

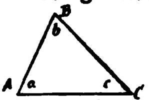

Fig. 11.

A geometrical figure bounded by three intersecting straight lines is called a *Triangle*. Thus, in Fig. 11, *ABC* is a triangle. Each triangle has three sides and three angles. The sum of the three angles of every triangle equals 180°; thus, if *a*, *b* and *c* are the three angles in the triangle *ABC*, then $a + b + c = 180°$. Therefore, if two angles are known in any triangle the remaining angle can be found; for instance: Angle $a = 70°$ and angle $b = 80°$; find angle *c*. Solution: $a + b + c = 180°$; substitute known values for $a + b$:

$$70 + 80 + c = 180.$$
$$150 + c = 180.$$

Subtract 150 from both sides:

$$c = 180 - 150.$$
$$c = 30°.$$

If in a triangle, two sides are equal, it is called isosceles

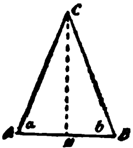

Fig. 12.

(See Fig. 12.) If $AC = CB$, and if the line CM is drawn so that $AM = BM$, then the line CM divides the angle ACB in two equal parts and stands perpendicular to AB at point M.

If in a triangle two sides are equal, the angles are also equal opposite these equal sides. Thus, if $AC = CB$, in Fig. 12, then angle b = angle a.

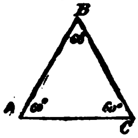

Fig. 13.

If in a triangle the three sides are equal to each other is is called an *equilateral* triangle, as in Fig. 13. If the side $AB = BC = AC$, then the angles must also be equal, hence angle a = angle b = angle c, and such a triangle is also called *equiangular*, so that an *equilateral* triangle is also *equiangular*.. Furthermore, since the sum of the three angles of every triangle equals 180°, then in the equiangular triangle each angle must be $180 \div 3 = 60°$. So, if the three sides in a triangle are equal, it tells at once that each angle in the triangle equals 60°, and if each angle in a triangle equals 60° it shows that the three sides are equal to each other.

If in a Triangle ABC, Fig. 14, the angle $ABC = 90°$, it is called a right angled triangle, and the side AB opposite the right angle, is called the Hypothenuse; hence, both of the other two angles must be acute angles, and as their sum makes 90°, they are also complementary angles.

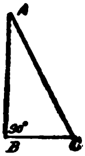

Fig. 14.

In any triangle the sum of two sides is greater then the third side.

In a right angled triangle the square of the Hypothenuse is equal to the sums of the squares of the other two sides. Thus, if in Fig. 14 $AB = 4$, $BC = 3$ and $AC = 5$, then $5^2 = 4^2 + 3^2$, or $25 = 16 + 9$. This is a very important principle and is referred to as the forty-seventh problem of Euclid. The fact that the three dimensions, 3, 4 and 5, or multiples thereof form right angled triangles, is made use of to plot right angles as follows: Suppose it is required to draw a perpendicular at point B to the line BC; let $BC = 30'$, then AB must be $40'$ and AC must be $50'$ to make the line AB stand perpendicular to BC; a closed string 120 feet long with knots 30, 40 and 50 foot apart would set out a right angle anywhere; also, smaller dimensions, for instance, 15', 20' and 25', or 9', 12' and 15' will do the same.

3. *Parallels and Quadrilaterals.*

A figure bounded by four intersecting straight lines is

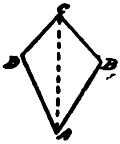

Fig. 15.

called a Quadrilateral; thus, *ABCD*, in Fig. 15, is a quadrilateral; it has four sides and four angles; a line like *AC* is called a ***diagonal***; this diagonal cuts the quadrilateral into two triangles which shows that the sum of the four angles of any quadrilateral equals 360° (two triangles).

If two lines, *AB* and *CD*, are cut by a third line *EF*, so that the sum of the inner angles $a + b = 180°$, then the lines *AB* and *CD* are called parallel, which means they will not intersect each other no matter how far they are extended. It is easily seen that angle $a =$ angle c, and angle $b =$ angle d as well when *AB* and *CD* are parallel.

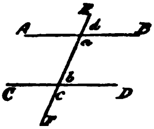

Fig. 16.

If in a quadrilateral the opposite sides are parallel it is called a parallelogram; thus, if in Fig. 17, *AB* is parallel to *DC* and *AD* is parallel to *BC*, then *ABCD* is a parallelogram and the opposite sides are equal; also, the opposite angles are equal; hence, $AB = DC$ and $AD = BC$; also, angle $a =$ angle c and angle $b =$ angle d; furthermore, if the two diagonals are drawn, *AC* and *BD* (See Fig. 18), intersecting in *O*, then $AO = OC$ and $BO = DO$, which means the diagonals bisect each other.

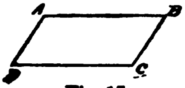

Fig. 17.

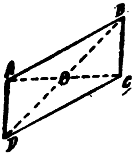

Fig. 18.

If each angle in a quadrilateral equals 90°, then it is called a rectangle or oblong (See Fig. 19.) All the properties of the parallelogram belong to this figure as well, and it has one special feature which is, that the diagonals in a rectangle are equal. Thus, $AC = BD$. This principle is made use of to test rooms or other rectangular areas to see whether they are square; for instance, let *ABCD* represent a room and let it be tested if it is square; take a string from the corner *A* to corner *C*, then take same string and stretch it from *B to D*; the string will fit both ways if the figure is a rectangle; if not, proves that the corners are not right angles. If in a rectangle all the sides are equal, it is called a square; thus, if in Fig. 20, $AB = BC = CD = AD$, then the figure is a square; all the properties of the rectangle apply also to the square with this additional one that the diagonals *AC* and *BD* intersect each other under right angles, and that the angles formed by the diagonals with the sides of the square are all 45°.

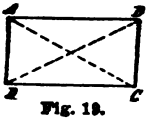

Fig. 19.

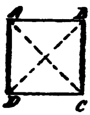

Fig. 20.

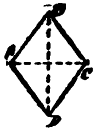

Fig. 21.

If in a parallelogram all the sides are equal it it called a *Rhombus*. Thus, if in Fig. 21, $AB = BC = CD = AD$, the figure is a Rhombus, and all the properties of the parallelogram apply to this figure with this one addition, that the two diagonals AC and BD stand perpendicular to each other.

If in a quadrilateral two sides are parallel, but the other two are not, like in Fig. 22, it is called a *trapezoid*, if AB and DC are parallel and the middle point M of AD is connected with the middle point N of BC, then the line MN is parallel to AB, and CD and $MN = \dfrac{AB + CD}{2}$

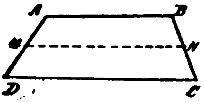

Fig. 22.

4. *Mensuration of Polygons.*

Any geometrical figure bounded by straight lines is called a *Polygon*. Any such closed figure embraces a certain part of the plane surface called area. Areas are measured by some unit area, which is the square whose side is the unit of length; thus, a square whose side is an inch, is called a square inch; the square whose side is 1 foot, is called a square foot; the square whose side is 1 meter long, is called 1 square meter, etc. If thus, one polygon is found to contain 8 square inches and another contains 16 square inches, then the former is half the size of the latter.

It is evident, that in a rectangle 2 inches wide and 4 inches high (See Fig. 23), the area will be 8 square inches, because by drawing the lines EF, GH, JK and LM from the inch points parallel with the sides of the rectangle the figure is cut into 8 equal squares, each equal to 1 square inch; hence, the area of a rectangle is found by multiplying the base AB, by the height BC. The area of a parallelogram is found also by multiplying the base by the height, but care must be taken to take the height correctly. Thus, in Fig. 24, if AB is taken for the base, then EF is the height, which is the perpendicular distance between AB and CD; if AD is taken as the base, then GH must be taken as the height.

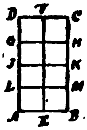

Fig. 23.

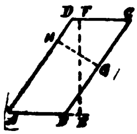

Fig. 24.

Any parallelogram can be divided into two

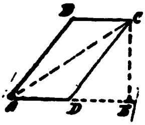

Fig. 25.

equal triangles, as in Fig. 25, where the line AC divides the parallelogram $ABCD$ into two equal triangles ABC and ADC, so each triangle is equal to half of the area of the parallelogram. Let CE be the perpendicular distance between AD and BC, then the area of the triangle $ACD = \frac{AD \times CE}{2}$, which gives the rule that the area of a triangle is found by multiplying base by height and divide by 2. Let in Fig. 26, the length of the three sides be a, b and c respectively, then each of them can be considered as base, and the right height is the perpendicular drawn to such base from the opposing vertex; if then three heights are drawn they pass through the same point O; let h be the height from A to a, h' the height from point B to b and h'' the height from point C to c, then the area of the triangle ABC can be expressed as follows: $\frac{ah}{2}$, $\frac{bh'}{2}$ and $\frac{ch''}{2}$, all of which give the same area and are therefore equal;

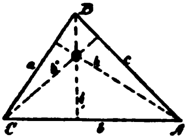

Fig. 26.

hence: $\frac{ah}{2} = \frac{bh'}{2} = \frac{ch''}{2}$.

Any polygon can be divided into triangles. In Fig. 27 is shown a heptagon (seven-sided polygon), $ABCDEFG$; by drawing the diagonals AC, AD, AE, AF, the polygon is cut into five triangles and by finding the area of them and adding them gives the area of the heptagon. Let $AC = a$, $AD = b$, $AE = c$, and $AF = d$; also, the height from B to $a = h$, to b from $C = h'$, to $c = h''$ from D and F to $c = h'''$, and from G to $d = h^4$, then the area A of the heptagon would be expressed as follows:

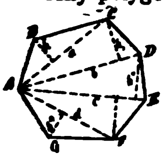

Fig. 27.

$$A = \frac{ah}{2} + \frac{bh'}{2} + \frac{ch''}{2} + \frac{ch'''}{2} + \frac{dh^4}{2}.$$

In this expression the factor ½ can be set out:

$$A = \tfrac{1}{2}\,(ah + bh' + ch'' + ch''' + dh^4).$$

This principle can be used for any polygon.

5. *Proportionality.*

Things of the same kind can be compared with each other numerically; thus, if a line *AB*, in Fig. 28, is 4" long, and a line *CD*, is 8" long, then *CD* is twice as long as *AB*, or *AB* is ½ of *CD*; this is called a ratio. Now, let *EF* be 3" and *GH* be 6" long, then the ratio of *EF* to *GH* is also the same as that of *AB* to *CD*; hence, the four lines shown in Fig. 28 form two equal ratios and can be combined into the proportion:

Fig. 28.

$AB : CD = EF : GH$.

(Read: *AB* is to *CD* as *EF* is to *GH*).

The four quantities are called the *terms* of the proportion; *AB* is the *First Antecedent*, *CD* the *First Consequent*, *EF* the *Second Antecedent* and *GH* the *Second Consequent*. In reality a proportion consists of two equal fractions. As above $AB : CD = 4/8$, and $EF : GH = 3/6$, or $4/8 = 3/6$.

The principal rule relating to proportions is that the product of the two extreme members is equal to the product of the middle (mean) members; so, if $3 : 6 = 4 : 8$, then

$3 \times 8 = 6 \times 4$.

This rule is used to find an unknown member in a proportion; for instance, in Fig. 28 let the length of the line *EF* be unknown and let it $= x$; then

$3 : x = 4 : 8$

form the product of the mean and extreme members;

$4x = 24$;

divide both sides by 4 gives $x = 6$.

6. *Similarity.*

Triangles are similar when the angles in the one triangle are respectively equal to the angles in the other triangle. In Fig. 29 the line *DE* is drawn parallel to *BC*, then angle $d =$ angle b, and angle $e =$ angle c, and the angle a is as well in the triangle *ADE* as in triangle *ABC*; so the triangle *ADE* is similar to triangle *ABC*, and the sides in the one triangle are proportional to the sides in the other which gives the proportions:

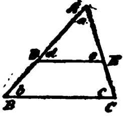

Fig. 29.

$AD : AB = AE : AC$; also,
$AD : DE = AB : BC$; also,
$AE : DE = AC : BC$, etc.

In picking out the proportional pieces care must be taken that it is done correctly, *i. e.*, that the pieces are similarly placed, so if *AD* is picked as the first antecedent and *DE* as the first consequent, then the second antecedent must be similar located in the other triangle; *AD* lies opposite the angle e and angle e = angle c, so the line opposite angle c corresponds to *AD*; this is *AB*; likewise, the first consequent *DE* lies opposite angle a; hence, the second consequent must have the same position in the other triangle which points to line *BC* lying opposite the same angle.

7. *Mean Proportionality.*

When in a proportion the two middle terms are equal it is called a mean proportion, for instance:

$a : b = b : c$,

which by forming the products of means and extremes gives the equation.

$b^2 = ac$.

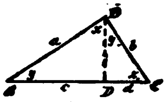

Fig. 30.

This principle is demonstrated in a right angled triangle *ABC*, Fig. 30, in which the line *BD* is drawn perpendicular from the right angled corner to the Hypothenuse; then there are three similar triangles; triangles *ABC* and *ABD* are similar; hence the sides are proportional; let $AB = a$, $BC = b$, and $AD = c$, $DC = d$, and $BD = h'$, then $: c + d : a = a : c$; also $c + d : b = b : d$. This means that the perpendicular from the right angle of a right angle triangle to the Hypothenuse divides it so that each of the other two sides is a mean proportional between the adjacent segment of the Hypothenuse and the Hypothenuse.

The first principle proves also the forty-seventh problem of Euclid, that the square of the Hypothenuse is equal to the sum of the squares of the other two sides as follows:

Form the product of the means and extremes:

$a^2 = c\ (c + d)$

$b^2 = d\ (c + d)$.

Add these two equations together:

$a^2 + b^2 = (c + d)\ (c + d) = (c + d^2)$;

but $c + d$ is the Hypothenuse.

As an example let $AB = 4$ ft., $BC = 3$ ft., and $AC = 5$ ft., then c and d can be found from the proportion:

$4 : c = 3 : 5 - c$, or

form the product of the means and extremes:

$3c = 4\ (5 - c)$.

$3c = 20 - 4c$. Add $4c$ on both sides:

$7c = 20$. Divide both sides by 7:

$c = 2.857$ ft.

then $d = 5 - 2.857 = 2.143$ ft.

To find the height $BD = h$ use the proportion:
$h : 4 = 2.143 : 3.$
$3h = 8.572.$
$h = 2.857$ ft.

To change the decimal fraction of the feet into inches, multiply by 12; thus: $12 \times .857 = 10.28$ inches. To change the decimal fraction of inches into the nearest 32nd of an inch, proceed as follows: Subtract 1 from the nearest 1/100, if fraction lies between 12/100 and 37/100; subtract 2 if fraction lies between 38/100 and 63/100; subtract 3 if fraction lies between 63/100 and 87/100, and subtract 4 when the fraction lies between 88/100 and 100/100; then divide by 3 and the nearest number in the quotient will be the answer in 32nds; thus, to change the 28/100 inches above into the nearest 32nd, subtract 1, which makes 27; divide by 3, which gives 9; so the answer is 9/32 inches. To change, for instance, .57 inches to the nearest 32nd, subtract 2 from 57, making 55, divide by 3 gives 18; so the nearest 32nd for .57 inches is 18/32 or 9/16. To change .81 inches to the nearest 32nd subtract 3 from $81 = 78$, divide by $3 = 26$, so $26/32 = 13/16$ is the answer. Also, to change .95 inches to the nearest 32nd, subtract $4 = 91$, divide by $3 = 30$; hence, $30/32 = 15/16$ is the answer. For fractions smaller than 12/100 no subtraction is made; for instance, to change 8/100, divide 8 by $3 = 3$; hence 3/32 inches is the answer.

8. *The circle and its properties.*

The circle is an uniformly curved line, each point of which has the same distance from a point within called the center. Thus in Fig. 31 let *O* be the center of the circle, draw any line *OA* to the circumference of the circle, then *OA* is called a radius; draw another radius *OB*, then $AO = BO$; the part of the circle line lying between *A* and *B* is called an arc, and the angle *AOB* is called a central angle, spanning arc *AB*; a straight line *AB* is called a chord. Equal chords span equal arcs and equal central angles. Thus, if *AB* and *CD* are equal, then the arcs *AB* and *CD* are equal, and the central angles *AOB* and *COD* are also equal. The area inclosed between two radii and an arc is called a sector, and the area lying between a chord and arc is called a segment. Thus sectors and segments of equal central angles or having equal arcs or chords in the same circle are equal. If a line *ST* is drawn

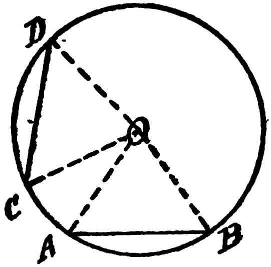

Fig. 31.

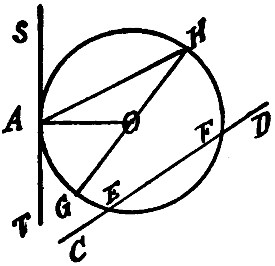

Fig. 32.

so as to touch the circle line in just one point *A* as indicated in Fig. 32, it is called a tangent, and if *A* is connected to *O* then *AO* stands perpendicular to *ST* in point *A*. A line like *CD* which cuts the circle line in two points *E* and *F* is called a secant. A chord *GH* passing through the center of the circle is called a diameter. An angle like *AHG* having its vertex in the circumference of the circle is called a periphery angle and is equal to ½ of the central angle *AOG*, both standing over the same arc *AG;* thus central angles are measured by the full arcs and periphery angles by ½ of the arcs upon which they stand. Any periphery angle standing over the diameter is consequently equal to 90°.

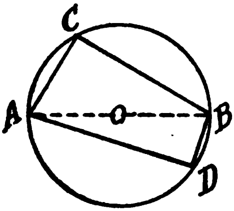

Fig. 33.

Thus in Fig. 33 if *AB* is a diameter then angle *ACB* = 90°; no matter which point of the circumference is connected with the ends of a diameter it always gives a right angled triangle as *ADB*.

The ratio of the circumference of a circle to its diameter is the decimal fraction 3.1416; this number, which is also called π (pi), is very important and enters in all circular problems, so that it should be known by heart; for ordinary work the fraction 3.14 or 3 1-7 may be used. Thus if the diameter of a circle is given the circumference is formed by multiplying the diameter by π; inversely the diameter of a circle is found by dividing the circumference by π. Thus let the diameter of a circle be 3 ft., then the circumference equals 3 × 3.14 = 9.42 ft. To find the diameter of a circle having a circumference of 10 ft., divide 10 by 3.14 = 3.184 ft.

The area of a circle is found by multiplying the square of the radius by π. To illustrate their relations by formulae let $\pi = 3.1416$, the circumference p the radius r and the area a, then the following formulæ obtain:

$p = 2\pi r.$ (1.)
$a = \pi r^2.$ (2.)
$r = p/2\pi.$ (3.)
$r = \sqrt{a/\pi}.$ (4.)

Applied Problems.

1. What is the circumference of a circle having a radius of 5 inches?

Solution. Substitute in formula 1 the given quantities, then: $p = 2 \times 3.1416 \times 5 = 31.416$ inches.

2. What is the area of a circle having a radius of 5 inches?

Solution. Substitute in formula 2 the given quantities: $a = 3.1416 \times 5^2 = 3.1416 \times 25 = 78.54$ square inches.

3. What is the radius of a circle, having a circumference 31.416 inches?

Solution. Substitute the given quantities in formula 3: $r = 31.416 \div 2 \times 3.1416 = 5$ inches.

4. What is the radius of a circle having an area of 78.54 square inches?

Solution. Substitute the given quantities in formula 4:

$r = \sqrt{78.54 \div 3.1416}$.

$r = \sqrt{25} = 5$ inches.

5. Find the diameter of a cast iron pipe having a cross sectional area of 32 square inches.

Solution. Here the area is given and the radius wanted, so use formula 4 and substitute the given quantities:

$r = \sqrt{32 \div 3.1416}$.

$r = \sqrt{10.186} = 3.191$ inches.

This makes the diameter $2 \times 3.191 = 6.382$ in., or 6⅜ inches.

5. Find the area of a Segment in a circle, Fig. 34, having a radius of 3 ft. and in which the central angle $ACB = 90°$.

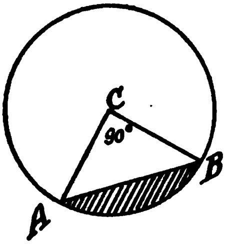

Fig. 34.

Solution. The area of the whole circle is equal to πr^2; hence the area of Sector $ACB = \pi r^2/4$; the area of triangle $ACB = r^2/2$, hence the area of the segment equals $\pi r^2/4 - r^2/2$; substitute given values:

$3.1416 \times 3 \times 3 \div 4 - 3 \times 3 \div 2$.

$7.0686 - 4.5 = 2.5686$ square ft.

ARTICLE IV.

TRIGONOMETRY.

1. *Principles and Definitions.*

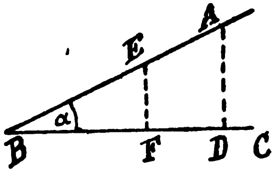

Fig. 35.

Trigonometry teaches how to compute the sides and angles of triangles. In Fig. 35, let *ABC* be any angle α and draw from any point *A* a line *AD* perpendicular to *BC*, then *ABD* is a right angled triangle; drop another perpendicular *E F* from any point *E*, then *E B F* is also a right angled triangle, and the two triangles are similar; hence:

$$EF : BE = AD : AB;$$

also $BF : BE = BD : AB$;

also $EF : BF = AD : BD$, and

$$BF : EF = BD : AD.$$

No matter where a perpendicular might be drawn parallel to *A D*, the above ratios remain the same so long as the angle α remains the same; it is also seen as soon as the angle α changes then the ratios of the sides change also; thus the ratio *EF/BE* changes for every degree of angular measure of α, and if a table be computed giving the value of this ratio for all the degrees of angle from 0° to 90° we can from such table draw conclusion as to the value of the angle if the sides *EF* and *BE* are known, or we can find the sides when the angles and one side are given.

These ratios of the sides of a right angle triangle are called trigonometric functions and are shown in a table embodied in this work.

2. *The Functions.*

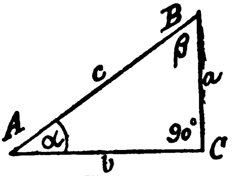

Fig. 36.

In Fig. 36 let the angle $ACB = 90°$, and let the sides of the triangle be *a*, *b* and *c* respectively, then the following ratios can be formed between the 3 sides:

a/c = sinus α, abbreviated sin α
b/c = cosinus α, abbreviated cos α
a/b = tangent α, abbreviated tg α
b/a = cotangent α, abbreviated cotg α
c/a = secant α, abbreviated sec α
c/b = cosecant α, abbreviated cosec α

The secant is the reciprocal of the cosine and the cosecant is the reciprocal of the sine, and both are but little used.

If the side *c* is = 1 then the side *a* is the sine and the side b is the cosine of α; in the relation to angle β the side a is the cosine and the side b the sine of β; since α and β are complementary angles it is seen that:

$$\sin\alpha = \cos\beta$$
$$\cos\alpha = \sin\beta$$
$$\operatorname{tg}\alpha = \operatorname{cotg}\beta$$
$$\operatorname{cotg}\alpha = \operatorname{tg}\beta$$

Thus it follows that for instant, sin 1° = cos 89°, or tg 2° 20′ = cotg 87° 40′, etc., hence the trigonometrical tables have been prepared so that every function for every angle is the cofunction of the complement.

3. *Transmutation of Functions.*

In Fig. 36, when $c = 1$, then a is the sine and b the cosine of α; and since $a^2 + b^2 = 1$, then

$\alpha = \sqrt{1 - b^2}$ and $b = b\sqrt{1 - a^2}$, so that

$\sin\alpha = \sqrt{1 - \cos^2\alpha}$ and

$\cos\alpha = \sqrt{1 - \sin^2\alpha}$

also $\operatorname{tg}\alpha = a/b = \sin\alpha \div \cos\alpha$

also $\operatorname{cotg}\alpha = b/a = \cos \div \sin\alpha$

also $\operatorname{tg}\alpha = 1/\operatorname{cotg}\alpha$ and

$\operatorname{cotg}\alpha = 1/\operatorname{tg}\alpha$

So to express the tangent a in sin α, substitute value of cosine as above:

$$\operatorname{tg}\alpha = \sin\alpha \div \sqrt{1 - \sin^2\alpha} \text{ or}$$
$$\operatorname{tg}\alpha = \sqrt{1 - os^2\alpha} \div \cos\alpha\text{; also}$$
$$\operatorname{cotg}\alpha = \cos\alpha \div \sqrt{1 - \cos^2\alpha}\, a$$
$$\operatorname{cotg}\alpha = \sqrt{1 - \sin^2\alpha} \div \sin\alpha.$$

4. *The Signs of the Functions.*

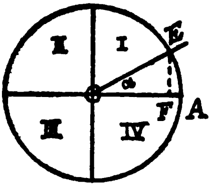

Fig. 37.

In Fig. 37 a circle is divided into 4 equal sections, called quadrants, let the radius *OA* equal 1 and let a moveable radius *OE* be journalled at point *O*, so that as it turns from the position *OA* upward through the 4 quadrants it will describe all the angles possible from zero to 360°; the angles in the first quadrant run from 0° to 90°; in the second quadrant from 90° to 180°, in the third quadrant from 180° to 270°, and in the 4th quadrant from 270° to 360°; but the trigonometrical functions are numerically the same in all the quadrants; thus if the angle $\alpha = 10°$ then its sine *E F* is also the sine of 170°, the sine of 190° and the sine of 350°, but the signs of the functions are differing in the different quadrants; hence the functions of the angles have been computed only for one quadrant.

The signs of the various functions are indicated in the following table:

Functions	Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
Sines	+	+	—	—
Cosines	+	—	—	+
Tangents	+	—	+	—
Cotangents	+	—	+	—

It is seen that all the functions are positive in the first quadrant and that in the second quadrant the sine is positive, but the cosine tangent and cotangent are negative.

The signs of the secant are the same as that of the cosine, and the signs of the cosecant the same as that of the sine.

5. *Numerical Limits of the Functions.*

By watching the moving radius in Fig. 37 it is seen that $E\ F = 0$ when $\alpha = 0$, so the sine of angle of $0° =$ = zero and grows as the angle grows, and reaches a maximum of 1.0 when $\alpha = 90°$; the cosine of $\alpha = 1$ when $\alpha = 0°$, and decreases as the angle increases, reaching 0 when $\alpha = 90°$. Likewise the tg $\alpha = 0$ when $\alpha = 0°$; the tangent grows as the angle grows and becomes infinitely large when $\alpha = 90°$.

The cotangent is the reverse of the tangent; when $\alpha = 0°$ the cotangent $= \infty$ (infinitely large), as the angle grows the cotangent decreases, and when $\alpha = 90°$ the cotangent $= 0$.

It is also seen that when $\alpha = 45°$, then the tangent and cotangent are each equal to 1 and the sine and cosine each equal to $\sqrt{.5} = .7071$. In a 60° triangle the cosine of an angle of $60° = \frac{1}{2}$.

6. *Application to Practical Problems.*

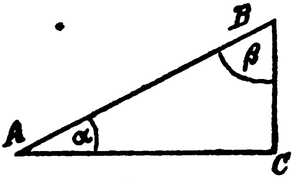

Fig. 38.

In the right angled triangle *ABC*, Fig. 38, let $AB = 17$ ft., $AC = 12$ ft.; find side BC and angles α and β.

Solution. The sine of the angle $\beta = AC/AB$ or $12/17 = .70588$; look in a table of natural sines and opposite .70587 you find 44° 23′, which is the right angle to the nearest minute; then since $\alpha + \beta = 90°$, then $\alpha = 90 - \beta$, or $\alpha = 90° - 40°\ 23' = 45°\ 37'$.

To find the side BC by trigonometry proceed as follows: $BC/AB = \sin \alpha$ then $BC = AB \sin \alpha$; look up the sine of $\alpha = 45° 37' = .71488$; then, $BC = 17 \times .71433 = 12.153$ ft.

Problem 2. The side $AB = 14.73$ ft. and angle $\alpha = 23° 16'$; find AC and BC.

Solution. Since $\alpha = 23° 16'$, then $\beta = 90 - 23° 16' = 66° 44'$; also: $AC/14.73 = \cos 23° 16' = .91868$; hence: $AC = 14.73 \times .91868 = 13.532$ ft.; $BC/14.73 = \sin 23° 16' = .39501$; hence: $BC = 14.73 \times .39501 = 5.8185$.

Problem 3. Let $AC = 13$ ft. and BC 20 ft., find the other part of the triangle.

Solution. $\operatorname{tg} \alpha = BC/AC = 20/13 = 1.53846$, look in a table of natural tangents and find that 1.53888 corresponds to 56° 59′ and 1.53791 corresponds to 56° 58′, so the value of the angle α lies between 56° 58′ and 56° 59′, a little nearer to 56° 59′, which will be taken for the answer.

Angle $\beta = 90° - 56° 59' = 33° 1'$.
The side AB is found as follows:
$BC/AB = \sin \alpha$;
$AB = BC/\sin \alpha$
Substitute the values for $BC = 20$ and $\sin 56° 59' = .8385$; then $AB = 20 \div .8385 =$
$AB = 23.851$ ft.

7. *Solution of Triangles.*

In Figure 39 let ABC be any triangle the length of the sides being a, b and c respectively and the angles being α, β and γ respectively the following relations obtain: $a : b : c = \sin \alpha : \sin \beta : \sin \gamma$; this means that in any triangle any two sides are to each other as the sines of the opposite angles. This rule is used to compute triangles when one side and two angles are given or when two sides and one opposite angle are given.

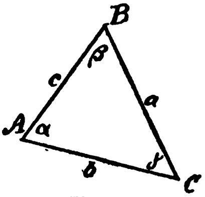

Fig. 39.

If two sides and the enclosed angle are given then the following formula is used:

$$a + b : a - b = \operatorname{tg} \frac{\alpha + \beta}{2} : \operatorname{tg} \frac{\alpha - \beta}{2}.$$

This means: the sum of two sides in a triangle is to its difference as the tangent of ½ the sum of the opposite angles is to the tangent of half the difference of the opposite angles. This is the second general formula.

The third general formula is expressed thus:

$$\text{Cos } \alpha = \frac{c^2 + b^2 - a^2}{2bc}$$

which means the cosinus of any angle is found by adding the squares of the two enclosing sides, subtract the square of the opposite side and divide by twice the product of the enclosing sides.

Problem 1. In Figure 39 let $a = 8$ ft., $b = 11$ ft and $\beta = 70°$. Find the side c and angles α and γ.

Solution. $a : b = \sin \alpha : \sin \beta$;
$b \sin \alpha = a \sin \beta$ and
$\sin \alpha = a \sin \beta / b$
Substitute the given quantities:
$\text{Sin } \alpha = 8 \times .9511 \div 11 = .6917$.

The angle corresponding to the sine .6917 in the tables is 43° 46′.

The angle $\gamma = 180 - \alpha - \beta = 180° - 72° - 43° 46' = 64° 14'$.

The side c is found as follows:
$c : b = \sin \gamma : \sin \beta$.
$c = b \sin \gamma / \sin \beta$; substitute known quantities:
$c = 11 \times .9006 \div .9511 = 10.416$ ft.

Problem 2. In triangle *ABC*, Fig. 39, side $a = 8$ ft, $b = 11$ ft. and angle $\gamma = 64° 14'$; find α, β and c.

Solution. $$a + b : a - b = \text{tg } \frac{\alpha + \beta}{2} : \text{tg } \frac{\alpha - \beta}{2}$$

the unknown term is $\text{tg } \frac{\alpha - \beta}{2}$; form the product of the means and extremes:

$$(a + b) \left(\text{tg } \frac{\alpha - \beta}{2}\right) = (a - b) \left(\text{tg } \frac{\alpha + \beta}{2}\right); \text{ hence:}$$

$$\text{tg } \left(\frac{\alpha - \beta}{2}\right) = (a - b) \left(\text{tg } \frac{\alpha - \beta}{2}\right) / (a + b);$$

Substitute the known quantities and remember if γ is subtracted from 180 it leaves $\alpha + \beta$:

$180 - 64° 14' = 115° 46' = \alpha + \beta$; then divide 115° 46′ by 2 $= 57° 53' = \frac{\alpha + \beta}{2}$, also since $b = 11$ and $a = 8$, angle β will be larger than angle α and the formula should be modified as follows:

$$\text{tg } \left(\frac{\beta - \alpha}{2}\right) = (b - a) \left(\text{tg } \frac{\beta - \alpha}{2}\right) / (b + a)$$

Hence: $tg\left(\frac{\beta-\alpha}{2}\right) = (11 - 8)\ (1.5931)\ /\ (11 + 8)$

$tg\left(\frac{\beta-\alpha}{2}\right) = 3 \times 1.5931 \div 19 = .2515.$

The angle corresponding to this tangent is found in the table of tangents to be: 14° 7′, thus:

$\frac{\beta+\alpha}{2} = 57^\circ\ 53'$; then $\beta + \alpha = 115^\circ\ 46'$

$\frac{\beta-\alpha}{2} = 14^\circ\ 7'$ $\beta - \alpha = 28^\circ\ 14'$

Add the two values together:

$2\ \beta = 114^\circ$ or $\beta = 72^\circ$;

then subtract $\beta - \alpha$ from $\beta + \alpha$:

$2\ \alpha = 87^\circ\ 32'$ or $\alpha = 43^\circ\ 46'$.

It will be seen if the 3 angles are added together the result is exactly 180°, which proves that the work is correct.

Problem 3. The 3 sides in a triangle are 28, 31 and 42 ft., find the 3 angles.

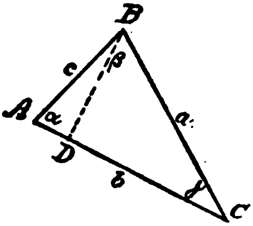

Fig. 40.

Solution. In Fig. 40 let a = 28 ft., $b = 31$ and $c = 42$ ft., then by using the third general formula we have:

$\cos \alpha = 31x31 + 42x42 - 28x28 \div 2x31x42$

$\cos \alpha = 1641 \div 2604 = .63031$ look in cosine table which gives the corresponding angles 50° 56′.

to find angle β use same rule:

$\cos \beta = 28x28 + 42x42 - 31x31 \div 2x28x42$

$\cos \beta = 2548 \div 4704 = .5417$

look for angle β in cosine table, 57° 12′

then angle $\gamma = 180 - (50^\circ\ 56' + 57^\circ\ 12') = 180 - 108^\circ\ 8' = 71^\circ\ 52'$.

ARTICLE V.

MENSURATION OF PLANE FIGURES.

Under mensuration is meant the determining of the areas of surfaces and the solid contents of bodies occupying space. In preceding articles reference has been made repeatedly to methods of finding the area of triangles, rectangles, etc. In the following are shown rules and application for the computing of areas of the more important geometrical surfaces.

1. *Triangles.* The area of a triangle is found by multiplying base by height and divide by 2. (See Art. 3.) When the 3 sides of a triangle are given, when a, b and c are the 3 sides and $S = \frac{1}{2}$ their sum, the area is found as follows: Area $= \sqrt{s\ (s - a)\ (s - b)\ (s - c)}$. To apply this formula let $a = 16$ ft., $b = 17$ ft., $c = 13$ ft., then $S = \dfrac{16 + 17 + 13}{2} = \dfrac{46}{2} = 23$. Substitute values in formula:

$$\text{Area} = \sqrt{23\ (23 - 16)\ (23 - 17)\ (23 - 13)}$$
$$= \sqrt{23 \text{x} 7 \text{x} 6 \text{x} 10} = \sqrt{9660} = 9.828 \text{ sq. ft.}$$

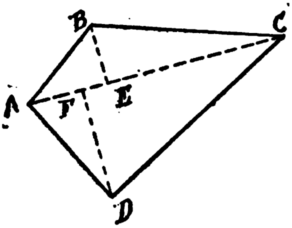

Fig. 41.

2. *Quadrilaterals.* The area of a rectangle, square, rhombus and parallelogram is found by multiplying base by height. The area of a trapez is found by cutting it into 2 triangles, finding the area of each of the triangles and adding them. Then in Fig. 41 the area of trapez *ABCD* is found by drawing *AC*, which divides the figure into two triangles and serves as a common base for both triangles; draw *BE* perpendicular to *AC* and *DF* perpendicular to *AC*, then area of triangle $ABC = \dfrac{AC \times BE}{2}$, and area of triangle $ACD = \dfrac{AC \times DE}{2}$, hence area of $ABCD =$

$$\frac{AC \times BE}{2} + \frac{AC \times DE}{2} = \frac{AC}{2}\ (BE + FD.)$$

3. *Polygons.* This principle may be extended to any polygon; thus for instance the 7-sided figure shown in Fig. 42 can be cut into 5 triangles by diagonals and the area of the figure will be the sum of the triangles. Another method can be used, indicated by dotted lines where the figure appears cut into 4 triangles and 3 trapezoids, all right angled. The advantage of the latter method is that the line AB serves as height for all the figures, and if the dimensions CC', DD', EE', FF', GG', and the dimensions on AB as AC', $C'D'$, $D'E'$, $E'B$, BF', $F'G'$ and $G'A$ are either given or observed, the area of the polygon $= \frac{CC'\times AC'}{2} + \frac{CC'+DD'}{2}\times C'D' + \frac{DD'+EE'}{2}\times D'E' + \frac{EE'\times BE'}{2} + \frac{BF'\times FF'}{2} + \frac{FF'+GG'}{2}\times F'G' + \frac{AG'\times GG'}{2}$; this can be simplified as follows:

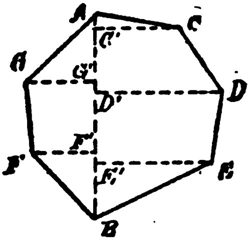

Fig. 42.

Area $= \frac{1}{2}\,[CC'\times AC' + C'D'\,(CC' + DD') + D'E'\,(DD' + EE') + BE'\times E'E + BF'\times FF' + F'G'\,(FF' + GG') + AG'\times GG'.]$

4. *Regular Polygons.* A regular polygon is a figure in which all the sides and all the angles are equal. The area of a regular polygon is easily found by cutting the same into triangles from the center point. Thus in Fig. 43 the regular heptagon is divided into 7 equal triangles by connecting the corners to the center O; then each central angle $= \frac{360}{7} = 51^\circ\ 25'\ 43''$. Draw OM perpendicular to AG; this bisects central angle so angle $AOM = 25^\circ\ 42'\ 51.5'' =$ say $25^\circ\ 43'$; let the side $AG = 12$ ft., then $AM = 6$ ft.; let $OM = h$ then $\frac{h}{6} =$ cotg

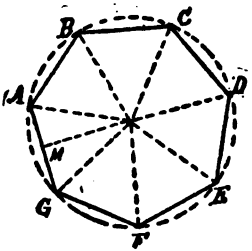

Fig. 43.

25° 43′, and $h = 6 \text{ cotg } 25° 43' = 6 \times 2.0763 = 12.4578$ ft.; area of triangle $AOG = 6 \times 12.4578 = 74.7468$ sq. ft., and area of heptagon $= 7 \times 74.7468 = 523.2276$ sq. ft.

5. *The Circle.* The circumference of a circle is 3.1416 times the diameter or, if r is the radius $2\pi r$. If the circle is considered a polygon with a very large number of sides, and the ends of these sides all connected to the center O, see Fig. 44, then the circle is cut up into a large number of equal triangles, all having the same height, which is the radius, and the sum of the base lines making the circle line; if the bases of triangle $= a$ the area of circle $= \frac{ar}{2} + \frac{ar}{2} + \frac{ar}{2} + \dots$ in which the factor $\frac{r}{2}$ may be set out; thus: $A = \frac{r}{2}(a + a + a + \dots)$ but the sum $a + a + a + \dots = $ circumference $= 2\pi r$, hence by substitution we get:

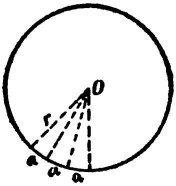

Fig. 44.

$$A = \frac{r}{2}(2\pi r);$$ this can be simplified to: $A = \pi r^2$ which, expressed in words means: The area of a circle is equal to the square of the radius multiplied by 3.1416.

6. *Elements of Sectors and Segments.* In Fig. 45 let AB be any chord, draw radii AO and BO and draw OM perpendicular to AB, then $AM = BM$ and arc $AC =$ arc BC. Let $r =$ radius of circle and $c =$ length of chord, then the other elements can be computed. Let $MO = x$, then $x^2 = r^2 - \left(\frac{c}{2}\right)^2$; or $x = \sqrt{r^2 - c^2/4}$.

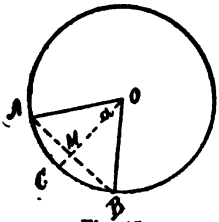

Fig. 45.

x can also be found without extracting the square root as follows: let angle $AOB = a$, then angle $AOM = \frac{a}{2}$.

$\sin\frac{a}{2} = \frac{c}{r}$ from which angle $\frac{a}{2}$ can be found; then, since $MO \div r = \cos \frac{a}{2}$ $MO = r \cos \frac{a}{2}$.

The area of sector AOB is found as follows: the area of the whole circle $= \pi r^2$; the area of a sector whose central angle $= 1°$ is $\frac{\pi r^2}{360}$, hence if there are a degrees in angle AOB the area of sector $AOB = \frac{\pi r^2 a}{360}$.

The area of segment ABC is found by subtracting triangle AOB from sector AOB, hence segment $ABC = \frac{\pi r^2 a}{360} - \frac{c}{2} r \cos \frac{a}{2}$.

The middle ordinate $CM = r - r \cos \frac{a}{2}$, or by simplifying: $CM = r \left(1 - \cos \frac{a}{2}\right)$.

Applied Problems. In a circle having a radius of 7′ 9″ is drawn a chord 10 ft. long; find central angle, area of sector, area of segment and middle ordinate.

Referring to Fig. 45, call central angle a, then $\sin \frac{a}{2} = 5 \div 7.75 = .64052$; refer to a table of natural sines, this gives $\frac{a}{2} = 39° \ 50'$, hence angle $a = 79° \ 40'$.

$$MO = r \cos \frac{a}{2} = 7.75 \times .76791 = 5.95131 \text{ ft.}$$

Then middle ordinate $= 7.75 - 5.95131 = 1.78969$ ft.

Area of sector $= 3.1416 \times 7.75 \times 7.75 \times 79.67 \div 360 = 41.765$ sq. ft.

Area of triangle $AOB = \frac{c}{2} r \cos \frac{a}{2} = 5 \times 5.95131 = 29.76655$ sq. ft.

Area of segment $ACB = 41.765 - 29.767 = 11.998$ sq. ft.

Problem. If in the preceding problem the angle a and radius r are given, then the chord, middle ordinate, arc, area of sector, area of segment can be computed. Let $a = 52°$ and $r = 200$ ft., then angle $AOM = 26°$ and let c = chord; then $\frac{c}{2} \div 200 = \sin 26°$ or $\frac{c}{2} = 200 \times .43837 = 87.674$ ft., hence $c = 175.348$ ft.

$MO = 200 \cos 26° = 200 \times .89879 = 179.758.$

Middle ordinate $= 200 - 179.758$ 20.242 ft.

The length of arc $= \frac{2\pi r a}{360} = \frac{\pi r a}{180} = 181.508$ ft.

The area of sector $= \frac{\pi r^2 a}{360} = \frac{3.1416 \times 200 \times 200 \times 52}{360} =$ 18151.47 sq. ft.

The area of triangle $ABO = 179.758 \times 87.674 = 15760.10.$

Area of segment $= 18151.47 - 15760.10 = 2391.37$ sq. ft.

ARTICLE VI.

MENSURATION OF SOLIDS.

1. *Prisms.*

A prism is a solid bounded by 2 parallel polygons as end surfaces and parallelograms as sides; if the end surfaces are parallelograms the solid is called a parallelopiped, and if it is bounded by 6 rectangles then it is called a right parallelopiped, and if in such a solid all faces are squares it is called a cube. Thus Fig. 46 represents a cube and if $AB = 1$ ft. the volume of the solid is called 1 cubic ft.; if $AB = 1$ inch the solid is called one cubic inch, etc. Such a cube is used as an unit to measure the volume of the other solids with.

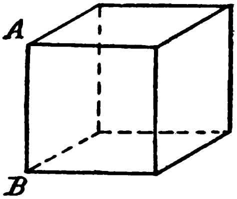

Fig. 46

The volume of a prism is found by multiplying the area of the base by the perpendicular height; the height is to be taken perpendicular between the two end surfaces. Thus in Fig. 47 the two prisms I and II contain the same volume if the bases have the same area and the heights are equal; observe the height in Prism II to be the perpendicular distance between the end surfaces.

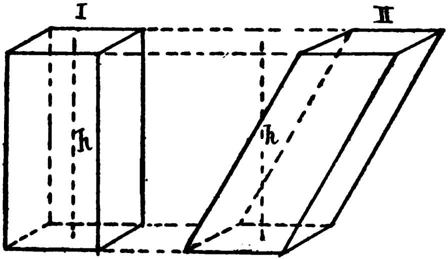

Fig. 47

Problem. Find the volume of a rectangular prism with a base 3′ × 4′ and a height of 5 ft. (Fig. 47-I.)

Solution. Let V = volume, then $V = 3 \times 4 \times 5 = 60$ cub. ft.

The surface consists of 6 rectangles, 2 end rectangles, each 3′ × 4′, or 12 sq. ft. each = 24 sq. ft.
4 sides, each 4′ × 5′, or 20 sq. ft. each = 80 sq. ft.

Total area in surface, 104 sq. ft.

2. *Pyramids.*

A pyramid is a solid having a plane polygon for its base and is bounded by as many triangles as the base has sides, all having a common vertex *P*; thus in Fig. 48 let *A, B, C, D, E,* be the base and *P* the vertex of the pyramid, then the base is a pentagon, and the other surfaces are 5 triangles with the point *P* in common; if a perpendicular is drawn from *P* to the base like *P H,* then *P H* is called the height of the pyramid.

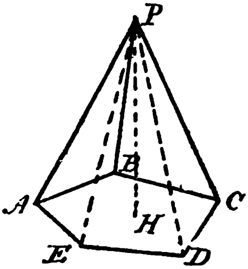

Fig. 48

Rule. The volume of a pyramid is found by multiplying the area of the base by the height and divide by 3. The area of the surfaces bounding a pyramid consists of the sum of the area of the base plus the areas of the triangles; thus in Fig. 48 the surface of the pyramid is equal to area of the pentagon *A B C D E* plus the 5 triangles, *A P B* + *B P C* + *C P D* + *D P E* + *E P A*

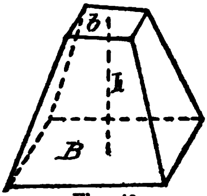

Fig. 49

If the top is cut away from a pyramid like in Fig. 49, it is called a frustum of a pyramid—if the area of the base = *B* and the area of the top = *b* and the height *h* (distance between top and bottom), then the volume of the frustrum $V = \frac{h}{3} (B + b + \sqrt{B\ b}).$

3. *Prismoids.*

In Fig. 50 is shown a solid known as a prismoid; it is a solid bounded by two parallel end surfaces and the sides are formed by trapezoids. The perpendicular distance *h* between the top surface *B* and the bottom surface *A*, Fig. 50, is called the height of the prismoid; if a section *M* is taken through the solid, midway between the two end surfaces, then the volume *V* of the solid is found by the following formula:

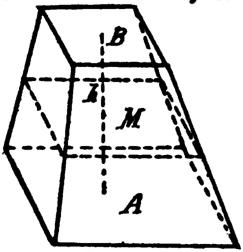

Fig. 50

$$V = \frac{h}{6} (A + 4\ M + B),$$ which expressed in words reads: The volume of a prismoid is found by adding 4 times the area of the middle section to

the sum of the top and bottom surface and multiply by 1/6 of the height.

Application. In Fig. 50, let A be a rectangle 3×5 ft., B a rectangle 2×4 ft., and let $h = 9$ ft.; find volume of prismoid.

Solution. The area of $A = 3 \times 5 = 15$; the area of $B = 2 \times 4 = 8$; the sides of the rectangle M are $\frac{3+2}{2}$ and $\frac{5+4}{2} = 2.5$ and 4.5 respectively; hence $M = 11.25$ and $4\ M = 45$; apply now formula $V = \frac{h}{6} (A + 4\ M + B.)$

$V = \frac{9}{6} (15 + 45 + 8) = 1.5 \times 68 = 102$ cub. ft. The surface of such a solid is equal to the sum of surfaces of the bounding side; thus let $S =$ the surface of the prismoid just considered, then $S = A + B + 2C + 2D$; where C and D respectively are the trapezoidal sides; care must be taken in figuring the surfaces not to confound the slant height of the trapezoids with the vertical height of the body.

4. *Cylinders.*

To find the volume of a cylinder multiply the area of the base by the height, because a cylinder can be considered a prism of a very large number of sides and therefore the volume of the cylinder is found by multiplying the area of base by height, just like the prism. When the two bases of a cylinder are circles with the centers perpendicular above each other, as shown in Fig. 51, it is called a right cylinder, and the line OO' is called the axis or height of the cylinder; if the radius of the cylinder is r and the height h, then $V = \pi r^2 h$.

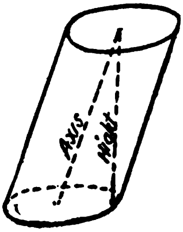

Fig. 52

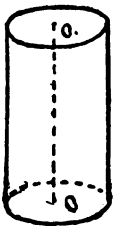

Fig. 51

The surface of the cylinder consists of the two end circles $= \pi r^2$ plus the shell, which is a rectangle h ft. high and the width of which is $2 \pi r$, hence $S = 2 \pi r^2 + 2 \pi r h$, which can be simplified to: $S = 2 \pi r (r + h)$; this can be expressed in words: The surface of a right cylinder is equal to the sum of radius and height multiplied by the circumference. To apply to a problem let the radius of a cylinder be 3 ft. and the height 5 ft.; find volume and surface.

$V = 3.1416 \times 3 \times 3 \times 5 = 141.372$ cub. ft.
$S = 2 \times 3.1416 \times 3 \times 8 = 6.2832 \times 24 = 150.81$ sq. ft.

Cylinders may be oblique, as shown in Fig. 52; then the height must be perpendicular to the two end surfaces for the figuring of the volume, but parallel to axis when figuring the area of surface.

5. *Cones.*

A cone, shown in Fig. 53, may be considered as a pyramid with a large number of sides; it has a base A which is a plane surface from which innumerable triangles reach to a common point V, called the vertex. A line VV', drawn perpendicular from the vertex to base is called the height of the cone; if the base A is a circle and VV' strikes its center it is called a right circular cone. The volume of such a cone is found as follows: $V = \dfrac{\pi r^2 h}{3}$ and the surface S consists of the base πr^2 plus the shell, which is a sector of a circle having a radius l and spans an arc $= 2 \pi r$; the central angle of this sector would be found as follows: full circle line is $2 \pi l$ ft. long; then one degree $= \dfrac{2 \pi l}{360} = \dfrac{\pi l}{180}$; as often as this is contained in the length of arc $2 \pi r$, that many degrees will be in the central angle; hence if x is the central angle then $x = 2 \pi r / \dfrac{\pi l}{180} = 360\, r/l$.

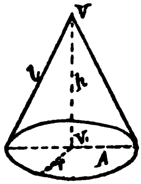

Fig. 53

The area of this surface is found from the following deduction: The area of a sector of 1° $= \dfrac{\pi l^2}{360}$, hence the area of a sector of $360\, r/l$ degrees equals $\dfrac{\pi l^2}{360} \times \dfrac{360 r}{l}$, which can be simplified to $\pi l r$; hence:
a circle having a radius l and spans an arc $= 2 \pi r$, hence:

$$S = \pi r^2 + \pi r l = \pi r\ (r + l.)$$

Problem. Find the volume and surface of a right circular cone when $h = 7$ ft. and $r = 2$ ft.

Solution. $V = \frac{\pi r^2 h}{3} = \frac{3.1416 \times 2 \times 2 \times 7}{3} = 29.321$ cub. ft.; for S first find $l = \sqrt{r^2 + h^2} = \sqrt{4 + 49} = \sqrt{53} = 7.283$, then $S = 3.1416 \times 2\ (2 + 7.283) = 58.327$ sq. ft.

The frustum of a cone will be handled like that of a pyramid, thus for a right circular cone: when R and r are the radius of the base and top and h the height:

$$V = \frac{h}{3} (\pi R^2 + \pi r^2 + \sqrt{\pi R^2 \times \pi r^2}),$$ which can be simplified:

$$V = \frac{h \pi}{3} (R^2 + r^2 + R\, r.)$$

Problem. Find volume of a frustum of a cone, if $R = 4$ ft., $r = 3$ ft. and $h = 5$ ft.

Solution. $V = \frac{5 \times 3.1416}{3} (5 \times 5 + 3 \times 3 + 5 \times 3.)$

$$V = \frac{5 \times 3.1416 \times 49}{3} = 256.564 \text{ cub. ft.}$$

6. *Sphere.*

A sphere is a solid, uniformly rounded so that each point in the surface has the same distance from a point within called the center; the distance from center to surface is called the radius. Every section made through a sphere forms a circle; when such a section passes through the center of the sphere it is called a great circle having a radius = to radius of sphere. The surface area of a sphere is equal to 4 times a great circle, thus if r = radius of sphere, then $S = 4 \pi r^2$.

The volume of the sphere may be considered as the sum of myriads of cones having their bases in the surface of the sphere and their vertices in its center, and as the volume of a cone is equal to base times 1/3 of height the volume of the sphere will be equal to the area of its surface multiplied by 1/3 of the radius, or $V = 4 \pi r^2 \times \frac{r}{3} = \frac{4}{3} \pi r^3$.

Example. Find the surface and volume of a sphere having a radius of 8 ft.

Solution. $S = 4 \pi r^2 = 4 \times 3.1416 \times 8 \times 8 = 804.25$ sq. ft.

$$V = \frac{4 \pi r^3}{3} = \frac{4 \times 3.1416 \times 8 \times 8 \times 8}{3} = 2144.94 \text{ cub. ft.}$$

If in a cylinder the height $= 2\,r$, its cross section becomes a square, Fig. 54; a circle of the radius r could be inscribed in it and represents the cross section of a sphere of the radius r, and the triangle shown represents a right cone having the same base and height as the cylinder; the relation of the volumes of these 3 bodies is as 1/3, 2/3, and 3/3, the cone being 1/3, the sphere 2/3 and the cylinder 3/3 of the whole, thus: Cylinder $= 2\,\pi\,r^3$, sphere $\frac{4\,\pi\,r^3}{3}$

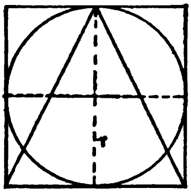

Fig. 54

and the cone $= \dfrac{2\,\pi\,r^3}{3}$

ARTICLE VII.

PRINCIPLES OF MECHANICAL FORCES.

1. *General Principles.*

Any cause which produces motion or changes motion is called a *force*. A force which acts for only one instance is called an impulsive force and produces uniform rectilinear motion; and a force acting constantly like the force of gravity is called an incessant force and produces accelerated or retarded motion. The principal features of mechanical laws are expressed by Newton as follows: "Every body continues in its state of rest or motion in a straight line unless acted upon by some external force."

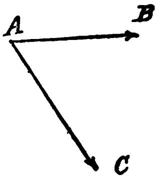

Fig. 55

"Every motion or change of motion is in the direction of the force acting and proportional thereto."

"To every action there is always an equal and opposite reaction."

Forces are represented by straight lines, the length of which indicates its magnitude, the direction of which gives the direction of the force by the arrowhead. Thus, in Fig. 55 *A B* represents a certain force acting upon the point *A* in the direction *A B* and with a strength proportionate to the length *A B*. If another force *A C* acts upon the point *A* in the direction *A C* and with a strength proportionate to its length; thus if *A B* is 3 Force Units and *A C* is 4-Force Units then the relation of the forces is fixed.

Forces are usually measured by the pound which is the pressure exerted by the mass of a substance weighing a pound. Other units are used, such as the gram, or kilogram, but in the United States the pound as a weight unit and the foot as a lineal unit are still used almost exclusively.

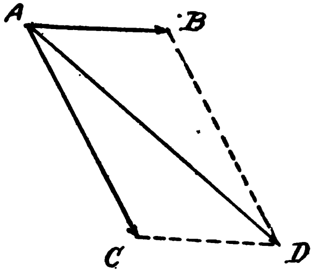

Fig. 56

2. *The Parallelogram of Forces.*

If two forces *A B* and *A C* act upon the point *A* under a certain angle then the point *A* cannot follow either of the two forces but will travel along a line *AD* which is the diagonal of a parallelogram formed about the two forces. The action of the two forces is then exactly the same as if a force *AD* alone was acting upon point *A;* such a force *AD* is called the resultant and *AB* and *AC* are called components and the arrangement is called the parallelogram of forces. This principle is used to find the resultant for any number of forces acting on the same point; when there are more than two forces it is called the polygon of forces.

One way of combining different forces acting on the same point *A* is shown in Fig. 57 where two rectangular axes *AX* and *AY* are drawn through the point *A;* given the forces *P, Q, S* and *T* and the angles α, β, γ and δ; by dropping perpendiculars from the ends of the forces to the *X* and *Y* axis each of the four given forces is decomposed into two forces acting rectangularly with each other; thus instead of *P* we have $P' = P \cos \alpha$ and $P'' = P \sin \alpha$; instead of *Q* we have $Q' = Q \sin \beta$ and $Q'' = Q \cos \beta$ instead of *S* $S' = S \cos \gamma$ and $S'' = S \sin \gamma$ instead of *T* $T' = T \sin \delta$ and $T'' = T \cos \delta$. So instead of the 4 original forces we have here 8 forces acting in rectangular axes; the four forces acting on the *X X* axis can be combined into one, namely, $P' + T' - Q' - S'$, which will give one force, this we will call R' acting upward; likewise the 4 forces on the *Y Y* axis can be reduced to one $= P'' + Q'' - S'' - T''$, which we will call R''; then the final combination is shown in Fig. 58, the two forces R' and R'' acting on point *A* under right angle; the resultant *R* is then the diagonal of the rectangle and $R = \sqrt{R_1^2 + R_{11}^2}$, also the directions of the Resultant is readily found, for, let angle $R\ AR_{11} = n$, then $\operatorname{tg} n = \dfrac{R_1}{R_{11}}$

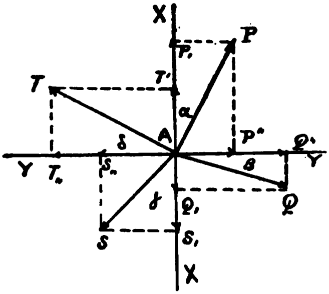

Fig. 57

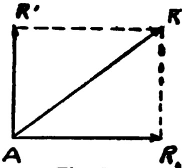

Fig. 58

3. *Parallel Forces Acting in the Same Direction.*

If two parallel forces *P* and *Q* tending in the same direction act upon a stiff bar *A B,* Fig. 59, then a resultant *R*

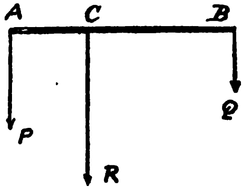

Fig. 59

may be set for P and Q, which is equal to the sum $P + Q$, also parallel to them and divides the distance $A\ B$ inversely as P and Q, that means if R acts at the point C then $A\ C : B\ C = Q : P$. This is a very important principle; it is seen if the point C be supported so it could not move then the two forces P and Q will be in equilibrium; the product of the force P into its distance $A\ C$ is called the moment of the force P, likewise the product of Q into the distance $C\ B$ is the moment of the force Q with reference to point C. This principle can be extended to any number of parallel forces. The force of gravity acting on a body can be considered as a large number of parallel and equal forces, each acting upon a particle. Thus if in Fig. 60 $A\ B$ is a line composed of a number of atoms A, B, C, D, E, F. G, H, then on each of them acts the force g downward, so for Ag and Bg we can set $Mg = Ag + Bg$, likewise for Cg and Dg we can set $g_1\ g_2$ acting at M, also for Eg and Fg we can set $g_2\ g_3$ acting at M and for Gg and Hg we can set $g_3\ g_4$, so that instead of all the parallel forces we have but one force acting in the middle point M. This point is called the center of gravity; so if $A\ B$ is a straight line or bar of uniform thickness its center of gravity is in its middle M, and if this point is supported then the whole bar will be in equilibrio, *i. e.*, will not move under the influence of gravity. It really acts as though the whole mass of the bar was concentrated in the center of gravity.

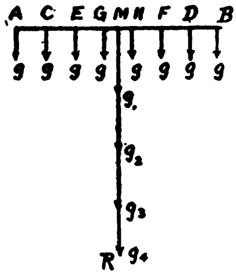

Fig. 60

4. *Center of Gravity for Various Shapes and Figures.*

The center of gravity of a line is in its middle.

The center of gravity of any symmetrical figure is in its geometrical center. Thus the centers of gravity of a parallelogram, rectangle rhombus and square are in the intersection point of the diagonals.

The center of gravity of a circle and the circle circumference is in the center of the circle.

The center of gravity of an ellipse lies midway between the two foci.

The center of gravity of a cylinder or right prism lies in the middle of its axis.

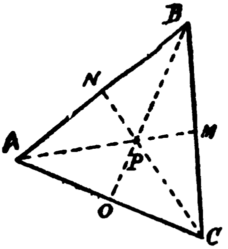

Fig. 61

The center of gravity of a homogenous sphere lies in its center.

The center of gravity of a triangle lies in the intersection of its medians, *i. e.*, lines drawn from the corners to the middle points of the opposite sides; thus if in Fig. 61 the lines *AM, BO* and *CN* are medians intersecting in point *P*, then *P* is the center of gravity of the triangle and is located ⅓ of the length of the median from the base.

Every body or shape has its center of gravity, and when it cannot be readily calculated it can be found experimentally as follows: Suspend the figures (see Fig. 62) by one corner *A* by sticking a pin through and having a thread *A P* with a little weight *P* attached, let the figure freely swing about the point *A*, and when at rest draw a light pencil line along *A P* over the figure, then the center of gravity must lie somewhere in this line *A P*.

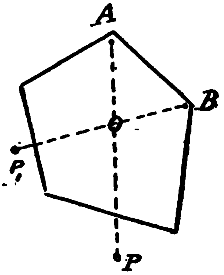

Fig. 62

Next hang up the figure from some other corner *B* and obtain another line *B P*, which intersects the line *A P* in *O*; then *O* is the required center of gravity.

The center of gravity of an arc of a circle is found as follows: let the length of arc *A B* = *a* and the radius *C M* = *r* and the length of chord *A B* = *c*, and let *g* be the distance of the center of gravity, *G* from *C* then $g = \frac{cr}{a}$ or in words: multiply the radius with the chord and divide by the length of the arc.

The center of gravity of every symmetrical body lies in its axis; thus the center of gravity of a sphere lies in its center, of a cylinder or regular prism in the middle of its axis, of a cone or regular pyramid upon the height ¼ of the distance from the base.

The center of gravity of the following circular figures lies on the axis of symmetry in each case; in the sector $2cr \div 3l$ from the center, of a semicircle .424 r from the center, of a quadrant .6 r and a segment $c^3 \div 12a$ from the center, r being length of a radius, c length of chord, l, center of arc and a = area of segment.

5. *The Laws of Gravity.*

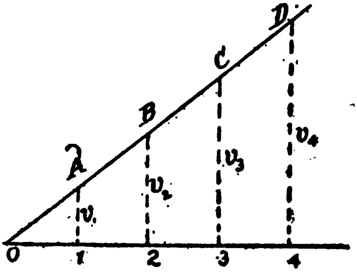

Fig. 68

The force of gravity when acting upon a free falling body produces an acceleration of 32.16 ft. per second, but the distance that a body falls the first second is 16.08 ft. This principle is illustrated in Fig. 63 in which the horizontal line indicates the time in seconds, and the ordinates v_1, v_2, v_3 and v_4 the velocities reached at the ends of the respective seconds, thus $v_1 = 32.16$ $v_2 = 64.32$, $v_3 = 96.48$, $v_4 = 128.64$, etc., the areas of the triangles represent the spaces fallen through; thus triangle 01A has an area of $\frac{1 \times 32.16}{2} = 16.08$, hence a body falls 16.08 ft. during the first second; the space fallen through in 2 seconds is represented by triangle 02B $= \frac{2 \times 64.32}{2} = 64.32$ ft.; the space fallen through in 3 seconds is represented by the area of triangle 03C $= \frac{3 \times 96.48}{2} = 144.72$ ft.; the space fallen through in 4 seconds is represented by the area of triangle 04D $= \frac{4 \times 128.64}{2} = 257.28$ ft. From these considerations the general law may be derived; if g is set for 32.16, then $v_2 = 2g$, $v_3 = 3g$, $v_4 = 4g$., etc., thus the velocity at the end of t seconds will be tg; now to determine the total space fallen through; the base of the triangle represents the time t and the height represents the velocity tg hence the area $= \frac{t^2 g}{2}$, so if h = space fallen through in t seconds h

$= \frac{t^2g}{2}$ and if V = velocity at the end of the time t then $V = tg = \frac{2h}{t}$, also $t^2 = \frac{2h}{g}$ or $t = \sqrt{\frac{2h}{g}}$, hence $v = g \sqrt{\frac{2h}{g}} = \sqrt{2gh}$.

These simple relations are very important as the force of gravity has to be dealt with everywhere, especially in hydraulic problems.

6. *Applied Problems.*

1. What is the velocity of a falling body at the end of the 7th second?

Answer: $7 \times 32.16 = 225.12$ ft.

2. What is the distance the body has fallen through in 7 seconds?

Answer: $h = \frac{t^2g}{2} = \frac{7 \times 7 \times 3216}{2} = 49 \times 16.08 =$ 787.92 ft.

3. How long will it take a body to fall 50 ft.?

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2x50}{32.16}} = \sqrt{3.1094} = 1.763 \text{ seconds.}$$

4. What is the velocity of a body after having fallen a distance of 50 ft.?

$V = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 50} = \sqrt{32.16} =$ 56.701 ft.

This checks the answer under 3, for if we multiply 1.763 by 32.16 we get 56.70 ft.

5. Through what distance must a body fall to acquire a velocity of 4 ft. per second?

For the solution of this problem use the formula:

$v = \sqrt{2gh}$ in which $g = 32.16$ and h = the required height; square both sides of the equation:

$v^2 = 2gh$

Divide both sides by $2g$:

$v^2/2g = h$

Substitute the given quantities:

$4 \times 4 \div 2 \times 32.16$; for ordinary problems it is near enough to use 32 instead of 32.16; hence:

$h = 16 \div 64 = \frac{1}{4} = 3$ inches.

This shows that after a body has fallen 3 inches it has acquired a velocity of 4 ft per second.

Summary of Formulæ is herewith given, referring to them in future by the letter given.

Let in all the formulae the following terms be used:

$g =$ acceleration of gravity $= 32.16$ ft. per sec.

$t =$ no. of seconds.

$v =$ velocity in ft. per second.

$h =$ height of fall in ft.; also termed the head when flowing water is considered, then

Formula (A) : $v = tg$
" (B) : $h = t^2g/2$
" (C) : $v = \sqrt{2gh}$
" (D) : $h = v^2 \div 2g$

These formulae establish the relation between velocity, time and height of fall.

ARTICLE VIII.

THE THREE STATES OF MATTER.

1. *General Principles.*

All things in Nature are either solids, fluids or vapors (gases), and some substances, like water, are known in all three states, namely in the solid form as ice and snow, as a fluid under normal conditions, and in the form of steam when heated to a certain point.

The state of a solid can then be defined that in a solid the molecules lie very near to each other so that is requires force to separate them; this proves the existence of an attractive force between the molecules, tending to pull the particles together; this force is called *cohesion.* It is assumed that there is also a repellant force active between the molecules tending to separate them; this force is called *repulsion.* Under the influence of these two forces the molecules are in a constant state of vibration; when a contraction of a substance takes place, due either to pressure or cold, then the cohesion increases, pulling the particles together; if heat is applied to a body the repulsion is increased and causes the expansion of the body. This expansion may become so great that the repulsion balances the cohesion and then the state of solid is changed to that of a fluid; in this state the body offers little resistance to the separation of its particles. If heat is continued to be applied the repulsion grows and overbalances the cohesion; then the state of the fluid changes to that of a gas, in which state the molecules tend to separate indefinitely and force must be applied to keep them together; thus, if sulphurated hydrogen is, for instance, set free by letting sulphuric acid act on iron sulphide, the gas will permeate at once the whole room, which is proved that it can be smelled all over the room.

Generally speaking, all substances can be transformed into the three states of matter by the agencies of heat or pressure, or both. Thus gold may be melted and even vaporized by using sufficiently high temperature; also oxygen gas may be made fluid by applying high pressure and cold, and may even be made solid by applying intense cold and very high pressure. The terms solids, fluids and gases apply to bodies that under ordinary conditions are solid, fluid or gaseous.

2. *Special Properties of Solids.*

a. Hardness. All solids offer a certain resistance to being broken. The Mohr's Scale of Hardness is used to designate the different degrees of hardness, namely: (1) Talc, (2) Selemite, (3) Calcite, (4) Fluorite, (5) Apatite, (6) Adularia, (7) Quartz, (8) Topaz, (9) Sapphire, and (10) Diamond. To test any substance for its hardness try to scratch with it the above named substances in succession; if it scratches, for instance, calcite but does not scratch fluorite its hardness is between 3 and 4 of Mohr"s Scale.

b. Malleability is the property of being widened out under the hammer, like lead or gold.

Ductility is the property of some substances to be drawn out in wires. The most ductile substance is platinum, which has been drawn into wires so fine that its diameter equals .00003 inch.

c. Tenacity is called the resistance different substances offer to being broken or ruptured. It is measured by the tensile strength of structural or building materials.

d. Elasticity is the tendency of bodies to assume their original form after having undergone a distortion; this property is also called *rigidity* when the body offers resistance only to distortion.

Forces which tend to produce alterations or distortions in bodies are called stresses; they are shearing, twisting and bending stresses. The bending stresses may be either compressive or tensile according to whether the forces tend to compress or elongate the fibres of the body under stress.

3. *Properties of Fluids.*

a. The surface of fluids at rest is horizontal. There being no rigidity between the molecules of a fluid their shape is altered by the least force so that when a fluid is at rest its surface is horizontal and it fills the form of the containing vessel. That the surface of a liquid must be at right angles to the force of gravity is seen from the following deduction; referring to Fig. 64, let BC be the surface of a fluid not at rest and let A be a particle of the fluid; the force of gravity AG pulls on A in a vertical direction and AG may be decomposed into AG_1, acting normally to the surface of the fluid, and AG_{11}, acting in a horizontal direction; the force AG_1 is neutralized by the body of the fluid but the force AG_{11} will pull point A down toward C and this will be kept up with all particles until the surface BC is normal to AG, or with

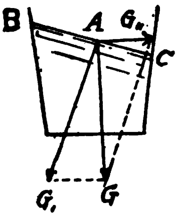

Fig. 64

other words until the surface of the fluid is level.

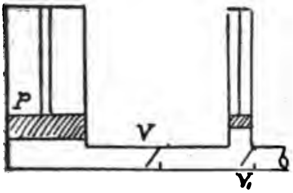

Fig. 65

b. The pressure in fluids is transmitted equally in all directions.

Fluids transmit pressure equally in all directions. An application of this principle is shown in Fig. 65, which represents a hydrostatic press, consisting of 2 cylinders in which pistons can move up and down; a valve v_1 admits water to the cylinders so that if piston p moves up valve v_1 opens and admits water; when p is forced down v_1 closes and v opens and the pressure which p exerts against the water surface is transmitted undiminished in all directions thus to the lower surface of the piston P, which rises and lifts the superimposed load. If the area of piston p is one square inch and the pressure forcing it down is 10 lbs., then the pressure exerted on the fluid is 10 lbs. per square; if the area of the piston P is 400 square inches then the total upward pressure on it will be 400 × 10 lbs. = 4,000; thus a force of 10 lbs. on piston p would hold a load of 4,000 lbs. on piston P in equilibrio, and a small excess of the force on p will make the piston P rise with its load. It is seen, however, that while a small force of 10 lbs. is lifting a load of 4,000 lbs. that the piston p has to work through 400 inches while piston P rises only 1 inch; so what is gained in power is lost in time.

c. The pressure depends on depth and area of surface.

In a vessel filled with a fluid at rest the pressure due to the weight of the fluid must be considered. Thus the pressure on the bottom of a vessel is equal to the weight of the liquid resting thereon. In Fig. 66 are shown 3 vessels, A, B and C, filled to the same height with a liquid; then the pressure per square inch on the bottom of A, B and C is equal per square inch. But if the area of bottom in vessel A is 2 square inches, in B 8 square inches and in C 1 square inch, then the pressure on the bottom of A is twice and in B is 8 times as great as in C, regardless of the amount of water in the various vessels. Also if the surface of the walls of a vessel is considered, the pressure

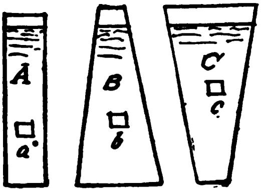

Fig. 66

as against the areas *a*, *b* and *c* is equal to the weight of a column of water having a base equal to the areas *a*, *b* and *c* respectively, and having a height reaching from the center of gravity of the section to the surface of the water.

d. Capillary Attraction.

This means if a tube is placed into a fluid which "wets" said tube, there is an attraction between the walls of the tube and the fluid and the latter rises in the former as indicated in Fig. 67. Let *AB* be a glass tube immersed in a tumbler containing water; then the water rises in *AB* above the level *CD* in the tumbler; this rise is proportional to the diameter of the tube and is greater the smaller the diameter is.

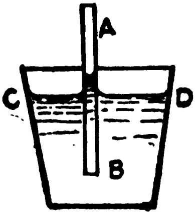

Fig. 67.

This phenomenon of capillary attraction is very important in nature. The growing of plants and trees is accomplished by the fibers and roots drawing their nourishment from the soil and sending it up to the various portions of the plant by virtue of capillary attraction.

Also the fact that a wick draws oil from a retainer up to the flame of the light is due to this fact, the wick acting as a number of capillary tubes. The evaporation of surface moisture in soils is caused by capillary attraction, as the pores in the soil act as capillary tubes and carry the moisture to the surface, when the dry air evaporates the water and the pores in the soil draw up further water. This effect is destroyed by breaking up the surface of the soil (mulching), which destroys the tubular texture of the soil and stops rapid evaporation. This principle is made use of in arid countries and is the principal factor in dry agriculture.

e. Capillary Repulsion.

When a tube is immersed in a fluid which does not "wet" the tube, a repulsion is taken place as indicated in Fig. 68. For instance, let *EF* be a glass tube immersed in quicksilver, then the height of the quicksilver in the tube will be lower than in the larger vessel, and the fluid is curving away from the walls of the tube and the vessel. The depression depends on the size of the tube and is greater the smaller the diameter of *EF*.

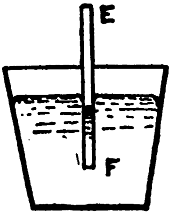

Fig. 68.

f. Buoyancy.

If a block of wood is placed into a vessel containing water it is observed that the block sinks down a certain depth and then remains suspended, which phenomenon is ordinarily termed *swimming* or *floating.* If instead of a block of wood a block of stone is used the stone will sink to the bottom of the vessel; when raising such stone under water it is found, however, that it seems much lighter, and in the act of lifting it from the water it appears to get heavier as it emerges from the fluid. Upon careful investigation it is found that every solid substance immersed in water loses as much weight as the water weighs which is displaced. This principle is called buoyancy, and applies to all fluids. If the weight of the displaced fluid is greater than the weight of the submerged body, as in the case of the block of wood, then body rises to a point so that the weight of the body equals the weight of the displaced fluid, and the body is then said to be floating. If the weight of the displaced fluid, however, is less than the weight of the immersed body, then the body will sink to the bottom of the vessel although its weight has been diminished equal to the weight of the displaced fluid.

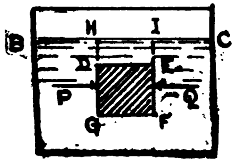

Fig. 69.

The principle involved is illustrated in Fig. 69, in which the vessel contains water to the line *BC*; if a block *DEFG* is immersed, then according to the law of pressures in fluids the pressure on top of the face *DE is* equal to the weight of the water column *DEHI;* the pressure on the bottom face is equal to the weight of the water column *GFHI,* hence the upward pressure on the face *GF is greater than the downward pressure on DE* by the difference *GFHI* — *DEHI* = *DEGF,* which is the weight of the displaced water. The side pressures *P* and *Q* are equal and opposite so they balance each other. The upward pressure, then, will, if it is greater than the weight of the block *DEFG,* lift the block until the upper part of it rises just high enough above the surface of the water so that the displaced water weighs exactly as much as the block; then the upward pressure equals the weight of the block and the downward pressure of the weight of the block equals the upward pressure of the water which produces equilibrium and the body floats.

If the weight of the block *DEFG* is, however, greater than the displaced water the force of gravity overcomes the upward pressure of the water and the body sinks to the bottom although with its weight diminished to the amount of the weight of the displaced water.

If instead of water other fluids are considered, the only difference there is is due to the density of the fluid, *i. e.,* the relative weight of the fluid compared with water. Thus if instead of water quicksilver is used, which is 13½ times as heavy as water, the buoyancy effect will be much greater, and a solid block of steel, which quickly falls to the bottom of a vessel filled with water, will float on quicksilver; on the other hand a block of wood floating on water will sink to the bottom of a vessel containing oil or alcohol.

g. Specific Gravity.

Different substances have different weights per cubic inch or cubic foot. A cubic ft. of water weighs 62.4245 lbs. at its greatest density of 39.1° F.; from this temperature it expands as it gets colder or warmer; at freezing point 32° F. 1 cubic ft. of water weighs 62.42 lbs., and at the boiling point, 212° F., it weighs 59.76 lbs; at 60° F. it weighs 62.37 lbs., which is a good average for perfectly pure water. The water we have to deal with, however, is seldom chemically pure, but contains other substances in solution which tend to make it heavier, and for this reason 62.4 lbs. is much nearer the average weight of a cubic ft. of water.

By specific gravity of any substance is meant a figure which indicates the ratio of the weight of a unit volume of that substance compared to the same unit volume of pure water. Thus the specific gravity of quicksilver is 13.5; this means that one cubic inch of quicksilver weighs 13.5 times as much as a cubic inch of pure water.

To find the specific gravity of any substance, first weigh the substance in air; let its weight in air equal W; next weigh the substance in water and let the observed weight equal w; let S equal the specific gravity sought, then:

$$S = \frac{W}{W - w}$$

To illustrate: It is required to find the specific gravity of a piece of ore; it is first weighed in air, and assume its weight is 12 ounces; next the same piece of ore is weighed under water, and we will assume its weight then is 10 ounces; then according to above formula find S by substituting the observed values:

$$S = \frac{12}{12 - 10} = \frac{12}{2} = 6$$

which gives 6 as the specific gravity of the substance in question.

This principle is very important and one easily applied. A table of specific gravities of the more important substances appears elsewhere in this book.

h. Compressibility of Water.

The common impression is that water is not compressible; it is, however, slightly so, the coefficient being .000003 per lb. pressure per square inch, so that it would take a pressure of a million pounds per square inch to reduce its volume 3/10. So for all practical purposes it may be said water is incompressible.

How little this compressibility of water does amount to may be inferred from this fact, that even at a depth of a mile in the ocean, where the pressure is something enormous, 5,280 × 625 = 333,000 lbs. per square foot, or nearly 2,292 lbs. per square inch, yet the weight of a cubic foot of water is only half a pound more than it is on the surface.

Hence it is perfectly permissible to ignore the compressibility of water.

4. Properties of Gases.

a. General deductions.

Gases are bodies in which the molecules possess no cohesion whatever, and tend therefore to separate from each other. Thus it requires force to keep the particles of any gas or vapor together, such as a closed vessel, or, as in the case of the atmosphere, the force of gravity, which prevents our air from leaving the earth.

b. Density of Gases.

All gases and vapors possess weight, but there is much difference in the weight of different kinds of gases. The lightest gas known is hydrogen, and as for fluids and solids, water is taken as the unit density, so for gases and vapors hydrogen forms the base and its density is called 1; then the densities of other gases are multiples; for instance, the density of oxygen is 16, of nitrogen 14, of carbon dioxide 22, etc. The density of air is 14.44, which means that one cubic ft. of air weighs 14.44 times as much as hydrogen.

c. The Pressure of the Air on the Surface of the Earth.

The atmosphere surrounds the earth like a mighty ocean many miles in thickness, and the weight of this mass of air presses on the surface of the earth. That this pressure is everywhere was first proven by Torricelli in the following manner. He took a glass tube *DC* (see Fig. 70), which was closed at D, filled it with quicksilver, closing the end *C* with his thumb, and immersed the end *C* under the surface of quicksilver contained in a vessel *AB*; then withdrawing his thumb the quicksilver in the tube dropped a distance until the distance to the surface *AB* of the quicksilver amounted to 30 inches; the fact that the quicksilver did not run out of the tube is accounted by the pressure of the air on the surface *AB*. And as the 30 inches of mercury just balance that pressure it shows that the pressure of the air on the earth is equal to that of a quicksilver ocean 30 inches thick, and as quicksilver is 13½ times heavier than water it is seen that if instead of quicksilver water would be substituted in the tube *DC*, then the pressure of the air on the surface *AB* would balance a column of water in *CD* equal to 13½ times 30 inches, or nearly 34 ft. This principle accounts for the air pressure lifting water in pumps and shows also the limits of such lift, the maximum of which at sea level would hardly reach 30 ft.

The air pressure diminishes rapidly in higher elevations because the thickness of the atmosphere decreases as well as the density, the density at sea level being ordinarily a maximum. On high mountains the density of the air and the air pressure is perceptibly less and pumps will therefore not lift water as high as in lower altitudes.

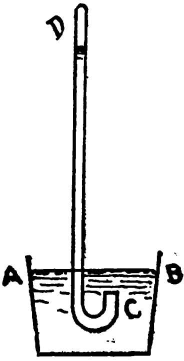

Fig. 70.

d. Compressibility of Gases.

If a bent glass tube *EC*, as in Fig. 71, with the short arm *C* closed, is filled up to the line *AB* with quicksilver, then there is a certain amount of air imprisoned in the short arm of the tube. If now quicksilver is added in *E* carefully, it rises in both arms, and when it reaches a point *D*, 30 inches above *C*, it is found that the original volume of the air in *BC* has been reduced ½, and likewise the pressure on the air has been doubled, as in addition to the original air pressure the weight of 30 inches of quicksilver has been added; if more quicksilver is poured into *E* until the top of it at *E* is 60 inches above the top of the quicksilver at *G*, then it will be found that the volume of the air in *G* is ⅓ of the original, and the pressure on that air is three times the air pressure. This experiment explains the important law of Boyle, which states that the volumes of gases are inversely proportional to the pressures; thus a cubic ft. of air under a pressure of 10 atmospheres (150 lbs. per square inch) will occupy only the space of 1/10 of a cubic ft. Inversely, when the pressure is relieved the gas expands without limit.

e. Buoyancy.

Bodies immersed in gases lose as much weight as the volume of the gas weighs which they displace. In this regard gases are similar to fluids, and the principle is vital to the navigation of air. Balloons filled with a gas lighter than air will rise in the air when the weight of the displaced air is greater than the combined weight of balloon and gas with which it is filled. As hydrogen is the lightest substance, it forms the most desirable gas for filling balloons as it has a greater buoyancy than any other gas.

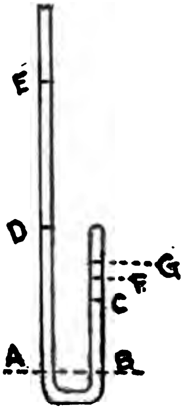

Fig. 71.

f. Evaporation in Air.

When the surface of air is in direct contact with fluids evaporation takes place, i. e., portions of the fluid assume the gaseous state and this continues until either all the liquid is evaporated or the air has become saturated. Thus the air can only hold a certain amount of the vapor of any particular fluid; this capacity is, however, increased by an increase in temperature.

ARTICLE IX.

GENERAL HYDRAULIC PRINCIPLES.

1. *The Cause of Flow in Fluids and Definitions.*

The flow of liquids is generally produced by the force of gravity. In Fig. 72 let the vessel *AGKL* be filled with water to line *AG* and let *EJ* be a pipe of considerable length, having tubes *C, D* and *B* in connection therewith, and tube *EJ* is closed at *J*. Then the water is at rest and stands level in the reservoir and in the tubes. If now the pipe is opened at *J* the water rushes out at *J* with a certain velocity; the water drops in the tubes *C, D* and *B* to points *C, D, B,* and if a line is drawn through these three points to the vessel *AG* it strikes at point *F,* a certain distance lower than the height of the water *GL.* The height *GF* is called the ***velocity head*** and is that part of the total potential energy of the height *GE* which is used to produce the velocity in the pipe; the remainder *FE* is called the ***pressure head,*** which is used in overcoming the friction throughout the pipe *EJ;* the height of the water in the tubes *C, D* and *B* show the decrease in the pressure heads successively; if the line *GH* is drawn parallel to *FB,* it will be seen that the velocity head *FG* is uniform throughout the flow length.

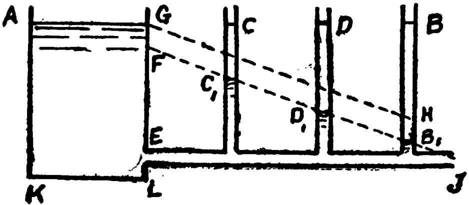

Fig. 72.

If the velocity is measured carefully at *J* it is found that the height *GF* is in excess of the height of fall required to produce such velocity. If v is the velocity at *J* then in the the equation: $v^2 = 2gh$

h can be found by dividing both sides by $2g$: $h = v^2/2g$; then suppose the velocity $v = 4$ ft., then $h = 16. \div 64 = .25$ ft. or 3 inches. The distance *GF* would be then 3 inches if there was no further loss in energy; there occurs, however, an additional loss at point *E,* which loss is termed the ***entry-head,*** and which depends on the form of entry and which may vary from one-half to one-tenth of the velocity head.

The line *FB* is called the ***hydraulic grade line.***

2. *Other Hydraulic Terms and Definitions.*

When water flows through a channel conduit or pipe the velocity of the flow depends on several factors—namely, the form of channels, the depth of flow, the roughness of the sides of the channel and the grade of the flow line, commonly called the sine of the slope. Thus if the cross-section of a channel is circular, as in Fig. 73, and if the water stands up to the line *AB*, then the wetted surface along the circumference from *A* to *B* via *C* is called the ***wetted perimeter;*** the ***flow area*** is the area of the cross-section of the flowing water and if the flow area is divided by the wetted perimeter the quotient gives the ***hydraulic radius,*** which will be termed r. Thus, if a pipe of the diameter d is flowing full the wetted perimeter is πd; the area will be $\pi d^2/4$; hence the hydraulic radius $r = \pi d^2/4 \div \pi d = d/4$.

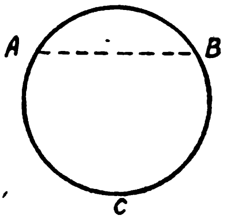

Fig. 73.

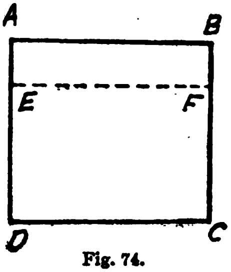

Fig. 74.

This proves that the hydraulic radius for all cylindrical conduits flowing either full or half full equals one-fourth of the diameter.

In Fig. 74 is shown a rectangular channel in cross-section *ABCD;* if the height of the flow reaches to *EF*, then the hydraulic radius will be found as follows: flow area equals $DC \times ED$ and wetted perimeter equals $ED + DC + FC$, hence:

$$r = DC \times ED \div (ED + DC + FC);$$

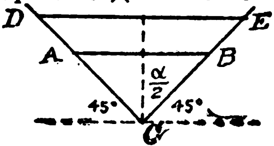

Fig. 76.

Should the box flow full, however, so that the water touches the upper surface *AB*, then the wetted perimeter increases greatly causing a decided drop in the hydraulic radius.

For instance, let the box be 2 ft. wide and 3 ft. high—then $r = 3 \times 2 \div (3 + 2 + 3 + 2) = 6 \div 10 = .6$.

On the other hand, let the water surface *EF* be 2 ft. high; then:

$r = 2 \times 2 \div (2 + 2 + 2) = 4 : 6 = .67$.

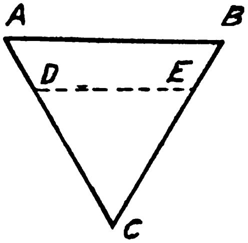

Fig. 75.

In Fig. 75 is shown a cross-section of a triangular channel. If the water stands up to line DE then the hydraulic radius r is equal to the area of the triangle DEC divided by the wetted perimeter $DC + EC$. Should the box, however, run full the wetted perimeter is increased by the length of the cover and results in a greatly decreased hydraulic radius.

3. *General Principles.*

The well-known formula of Kutter:

$$v = C\sqrt{rs}$$

is at the present time without doubt the most generally used and gives correct results in practice if applied rightly. The various elements entering into its composition must, however, be correctly determined and infelligently handled. In the above equation v is the mean velocity of the liquid in feet per second, r is the actual hydraulic radius and s is the sine of slope. The coefficient C is a complex quantity and is derived by calculation from the following formula:

$$C = \frac{41.6 + \frac{1.811}{n} + \frac{.00281}{s}}{1 + \left(41.6 + \frac{.00281}{s}\right)\frac{n}{\sqrt{r}}}$$

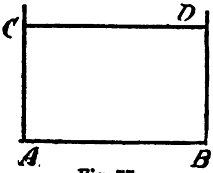

Fig. 77.

In this formula the quantity n is called the coefficient of roughness, s is the sine of the slope and r the hydraulic radius as explained above.

The factor n, then, is really the most difficult quantity to determine, as its correct selection is very important. There is, however, sufficient experience to be drawn from, so that the degree of roughness or factor n can be selected with considerable precision.

In this book the coefficient C has been calculated for factors of roughness varying from .008 to .050, which embraces all imaginable practical channels. Since the variation of the factor C is slight for very considerable differences in slopes, the factor C has been determined only for six different slopes which covers the field fully as any intermediate slopes can be easily interpolated when necessary. This will later be shown by practical problems.

The tables have been computed for the square root of r instead of r, as this gives a considerable wider range of application, and the factor C is found with sufficient accuracy by interpolation between the values shown.

4. *Forms of Open Channels.*

The form of a channel is an important factor for the flow of water, and we will consider the characteristics of some of the most important ones. All other things considered equal, the velocity of flow grows as the hydraulic radius r grows, which is plainly seen from the fundamental formula:

$$v = C\sqrt{rs}$$

hence the larger r the better the flow conditions will be. To compare the relative efficiency of various forms we will take the triangular, rectangular, trapezoidal and circular sections which are in general use. Let Fig. 76 show the cross-section of a triangular channel, the sides of which incline 45° to the horizon and form an angle of 90° with each other. This kind of section is best adapted when a considerable velocity of flow is required with a small volume of fluid. To prove this it is easily seen that the factor r grows or falls in a constant ratio, no matter how high the water rises or how low it goes; thus if the water stands up to line AB, the hydraulic radius r equals: area of triangle ABC divided by the wetted perimeter $AC + BC$; let $AC = a$, then area of triangle $= a^2 \div 2$; wetted perimeter $= 2a$, hence:

$$r = a^2 \div 2 \div 2a$$
$$r = a^2 \div 4a$$
$$R = a \div 4$$

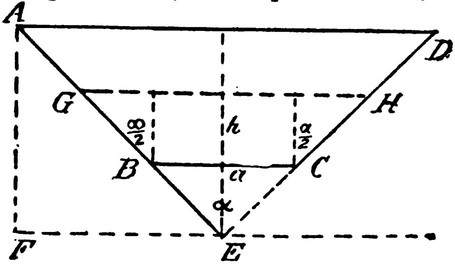

Fig. 78.

If a now is doubled, r is doubled; if a is made half, then r is made $r/2$, the general form of r being $a/4$, a being length of one wetted side.

The angle ACB may have a different value x; then, if h is the depth of flow, the area of flow equals $h^2 \operatorname{tg} \frac{x}{2}$, and the wetted perimeter $= \frac{2h}{\cos\frac{x}{2}}$, hence $r = h^2 \operatorname{tg} \frac{x}{2} \div \frac{2h}{\cos\frac{x}{2}}$, which can be reduced to $.5h \operatorname{tg} \frac{x}{2} \cos \frac{x}{2} = .5h \sin \frac{x}{2}$, which is the general form for r in triangular channels.

Fig. 77 is a rectangular section. Let CD be the upper plane of flow, then area $ABCD \div (AC + AB + BD)$ would be r. It is seen that when $AC = AB$ the hydraulic radius $= AB \div 3$; for if $AB = a$, area $= a^2$, and wetted perimeter $= 3a$, hence $r = a^2 \div 3a = a \div 3$.

With the width of channel constant, the hydraulic radius varies with the height; as stated above, it is $a \div 3$, when the cross-section of flow is a square; r decreases with the height and equals zero when the height AC is zero; r increases as the height increases, and reaches its maximum when AC equals infinity, in which case $r = a \div 2$.

The trapezoidal form of channel is shown in Fig. 78. If the two sloping faces AB and CD are extended until they meet in E, the section may be considered triangular with the lower part BCE cut away.

A section of this form will be useful when the variation in volume of flow is not very large, and where the cutting might have to be carried to too great depths if the true triangular form were adopted.

Best conditions are obtained if angle $AED = 90°$ and angle $AEF = 45°$. This would make the slope 1 to 1, which might have to be modified for the various conditions under which the channel is to be constructed.

Assume angle $AEC = 90°$ and $BC = a$; let GH be flow line and $h =$ depth of flow; then area of flow: $(a + h)h$; the wetted perimeter $P = GB + BC + CH$; $GB = HC = \sqrt{2h^2} = h\sqrt{2}$; hence wetted perimeter $p = a + 2h\sqrt{2}$.

If we divide p into area we get $r =$

$r = h\,(a + h) \div (a + 2h\,\sqrt{2})$.

It is seen from this formula that r increases as h and a are increased; if a is made equal to zero the formula changes:

$R = h^2 \div 2h\sqrt{2} = h \div 2\sqrt{2}$.

This is the case of the triangular section expressed in terms of the depth of flow. When h and a are equal the formula of r appears thus:

$r = h(h + h) \div h + 2h(\sqrt{2}$

$r = 2h^2 \div h(1 + 2\sqrt{2})$, which may be simplified to:

$r = 2h \div 1 + 2\sqrt{2}$

$r = h \div 1.914 = .52h$.

This is equal to more than half of the height and is the best section for high velocity of this kind.

If the angle $AED = x$, then the flow area will equal $h(a + h \,\mathrm{tg}\, \frac{x}{2})$ and the wetted perimeter $= a + \frac{2h}{\cos\frac{x}{2}}$, hence

$$r = \frac{h\,(a + h \,\mathrm{tg}\, \frac{x}{2})}{a + \frac{2h}{\cos\frac{x}{2}}}$$

which is the general form for the trapezoidal channel.

5. *The Circular Form.*

Fig. 79 shows the circular form of conduit and while it is a form in very extended use and is easily applied and figured when flowing either half full or full it is very difficult and tedious to determine the hydraulic radius and conditions of flow when the channel flows neither full nor half full. This difficulty becomes apparent if we consider a random height of flow as indicated by the line AB in Fig. 79. If the cylinder is just full, or just half full, it is easy to determine the wetted perimeter and area, and consequently the hydraulic radius, in the first case, the full area is divided by full circumference, and, in the second case, it is half area of circle divided by half the circumference. In either case $r = d \div 4$ if d is diameter of circle; this is easily proven, for the area of circle $= \pi d^2 \div 4$ and the circumference $= \pi d$, hence $r = (\pi d^2 \div 4) \div \pi d$.

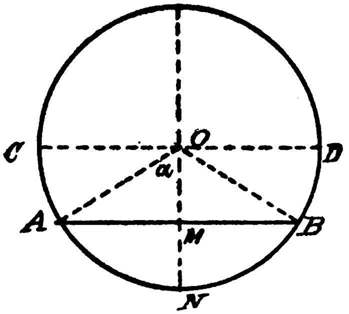

FIG. 79

Cancelling πd we retain:

$r = d \div 4$.

In the case of the semicircle we have: Area $\div \pi d^2 \div 8$; circumference $\pi d \div 2$; hence $r = (\pi d^2 \div 8) \div (\pi d \div 2) = d \div 4$.

Assume a typical case, taking the surface of fluid at AB with a certain distance below the center; here, if the versed sine MN is given, it will be quite a little problem to determine the area of the segment *ANB;* it requires still more figuring to get the length of the wetted perimeter *ANB;* thus, if $d =$ diameter of circle and $m =$ versed sine, then $OM = d/2 - m$ and $\cos \alpha = (d/2 - m) \div d/2$; this gives the center angle of the sector, as angle $AOB = 2\alpha$ and area of segment *ANB* can be found by subtracting triangle *AOB* from sector *OANB,* thus:

Area of sector: $\dfrac{\pi d^2 \alpha}{4 \times 360}$

Area of triangle: $AM \times OM$

$AM = d/2 \sin \alpha$ and $OM = \dfrac{d}{2} - m$, hence area of triangle $= \dfrac{d}{2} \sin \alpha \left(\dfrac{d}{2} - m\right)$ and flow area $a = \dfrac{\pi d^2 \alpha}{4 \times 360} - \dfrac{d}{2} \sin \alpha \left(\dfrac{d}{2} - m\right)$.

The circumference or wetted perimeter is found:

$p = \pi d \alpha \div 360$.

Divide a by p gives r, but it is seen that it requires considerable figuring.

Certain problems in hydraulics require many trial assumptions before a satisfactory solution is reached which multiply the work many fold. This the author has sought to avoid by the calculation of a number of tables which are embodied in this book.

The value of r grows with the depth of flow or the versed sine from zero to a maximum when m (the versed sine) equals $.813d$; hence the velocity of flow in a circular channel is greatest when .813 full; the quantity Q, however, is greatest when $m = .949d$, when the product of flow area times velocity reaches a maximum.

6. *Summary Comparisons.*

Summarizing the foregoing deductions the following facts may be stated:

1. For open channels where there is considerable variations in the volume of flow and where the best velocity is desired for small flow the triangular section is best.

2. For open channels of nearly constant flow the trapezoidal section is best, providing height of flow is equal to or greater than width of channel at bottom.

3. For closed channels flowing full or under pressure the cylindrical section is best.

7. *A New Form of Channel.*

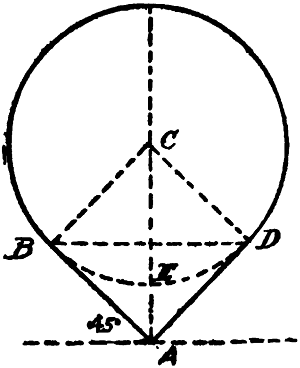

FIG. 80.

For sewers or other channels which have to accommodate a small dry weather flow and a considerable flow of storm water at times a combination section is suggested as indicated in Fig. 80; the lower part being formed by two planes forming a right angle with each other and the upper part being a part of a circle. Let it be a horizontal plane passing through vertex *A*, then angle α should be 45° and angle $BAD = 90°$. The size of such a sewer would be governed by the radius of the cylindrical part, as $AB = BC$.

Let $R =$ radius, then area will be:

$$a = \frac{3\pi R^2}{4} + R^2; \quad a = \frac{3\pi R^2 + 4R^2}{4}$$

$$a = \frac{R^2}{4}\ ((3\pi + 4); \quad a = 3.3562R^2.$$

The wetted perimeter flowing full is $p = \frac{2\pi R \times 3}{4} + 2R.$

Simplifying this expression we get:

$$p = \frac{6\pi R + 8R}{4} = \frac{R}{4}\ (6\pi + 8)$$

$p = 6.7124R.$
Hence: $r = a \div p;$ substitute values:
$r = 3.3562R^2 \div 6.7124R.$
$r = R \div 2.$

From this it appears that the hydraulic radius for this combination section is precisely equal to that of the full circle. Consequently when flowing full it develops the same velocity as would be done in a true circle, but its capacity is increased in the ratio of 3.3562 to 3.1416. The main advantage in this shape of a sewer or conduit is the better flow condition it affords in dry weather, when the volume of flow may be as low as one-twelfth of. the maximum capacity. The sewer is deeper by EA than the circular sewer,

$CA = \sqrt{2R^2} = R\sqrt{2}$ and since $CE = R$
$AE = R\sqrt{2} - R = R\ (\sqrt{2} - 1)$
$AE = .4142R.$

This makes total height of sewer $2.414R$, as against $2R$ for true circle.

This form should prove far more desirable than an elliptical shape, and with the general adaptability of concrete construction now-a-days should be easily built; it will be referred to later in order to show its superiority over other forms of sewers.

ARTICLE X.

THE COEFFICIENT OF ROUGHNESS.

In the determination of the factor C in Kutter's formula all the quantities entering into consideration are definite and fixed, except the coefficient of roughness n. It is therefore quite important that this value n be properly selected. The quantity n should represent everything which acts as resistance to the flow of water. Many thousands of gaugings have been made to determine the value of n in different kinds of channels, and the results obtained are sufficient to act as a guide in the selection of the proper factor for any proposed channel. Some gaugings made show n as low as as .007 and as high as .050, so that all variations in conditions are covered by this amplitude. In ordinary practice n as low as .007 is impracticable and this book begins with $n = .008$ as the lowest coefficient of resistance, which applies only to the smoothest glass pipe or polished metal pipes, perfectly straight and uniform.

Channels to be grouped under $n = .009$:

Glass pipe, lead pipe, tin pipe, straight and regular; wrought iron pipe, galvanized pipe, straight and regular; very hard wood, surfaced, straight and regular.

$n = .010$.

New cast iron pipe free of imperfections, new tin or lead pipe, all of perfect cross sections; neat cement pipes of regular cross cross sections and perfectly formed; flumes lined with evenly planed boards of straight line and uniform cross sections.

$n = .011$:

Cast iron pipe (new), neat cement channel, straight and regular; glazed tiling; very smooth planed plank channels of uniform section.

$n = .012$:

Used cast iron pipe, channels formed of two-thirds cement and one-third fine sand, planed boards in flumes of regular section.

$n = .013$:

Very smooth brick work plastered with cement, planed common boards of slightly varying cross-section, hard brick, smooth surface with clean joints, concrete channels, very regular and smoothly plastered.

$n = .014$:

Channels formed of regular masonry, regular brick work with slight variations in cross-section straight line, concrete channels fairly smooth or fairly well plastered.

$n = .015$:

Rough brick work, Ashlar masonry channels, small irregularities in cross-section and line; also channels of very rough concrete.

$n = .017$:

Channels of very rough brick work, or very rough masonry, very good ditches of uniform cross-section and smooth, paved channels.

$n = .020$:

Channels and rivers with paved banks and bed of fine sand (Ohio river, Point Pleasant, .021), ditches cut fairly smooth and uniform.

$n = .025$:

River beds, side walls of good Rubble masonry, channels of fair line and fairly regular cross-section, ditches of irregular section, (not too much so), with some small detritus or light growth of aquatic plants.

$n = .030$:

River bed with embankment walls, detritus in small quantities, pebbles and small boulders in bed, channels and ditches of irregular outline and grade channels with heavy growth of aquatic plants.

$n = .035$:

Channels, rivers and ditches with irregular bed and irregular side walls, boulders, heavy detritus, great variation in the slope and cross-sections.

$n = .040$:

Rivers, canals and ditches with irregular beds, coarse detritus, irregular cross-section, irregular slopes.

$n = .050$:

Rivers of very irregular beds, very coarse detritus, very irregular cross-section with large obstructions and very steep grades.

It is to be understood that the character of a channel has a great deal to do with the flow of water and it may be that the flow conditions of one kind of cast iron pipe may be covered by $n = .010$, while another may require $n = .012$. Similar variations must be looked for in brick, mason and concrete work, and the observant hydraulic engineer or irrigator must judge for himself what coefficient will be suited best for his particular work. Here is where enters the personal equation of the practical side, which cannot be covered in the book, but which each engineer must solve for himself. The foregoing list of different materials and channels are cited merely for an approximate guide.

ARTICLE XI.

HOW TO CALCULATE "n".

It is evidently of importance to be able to compute the coefficient n for any case where proper gaugings and observations have been made so as to define more clearly the different degrees of roughness. By an intricate transmutation of the general formula for C, given in Article IX, No. 3, the quantity n may be developed, and here follows the formula which results:

$$n = \sqrt{\frac{1.811\sqrt{r}}{BC} + \frac{r}{4}\left(\frac{C-B}{BC}\right)^2} - \frac{\sqrt{r}}{2}\left(\frac{C-B}{BC}\right)$$

In this formula r is the hydraulic radius

$s =$ slope

$$B = 41.6 + \frac{.00281}{s}$$

$C = v \div \sqrt{rs}$

$v =$ mean velocity.

Following is an application of this formula, illustrated upon a practical problem: A brick conduit 4 feet in diameter is flowing full; the sine of slope $= .0004$ and the observed mean velocity $= 2$ feet; find n.

Solution. $r = 1$ and $\sqrt{r} = 1$; $\sqrt{s} = .02$; hence:

$C = 2 \div .02 = 100.$

$B = 41.6 + .00281 \div 0004 = 41.6 + 7.02 = 48.62.$

Substitute these quantities in above equation:

$$n = \sqrt{\frac{1.811 \times 1}{48.62 \times 100} + \frac{1}{4}\left(\frac{100-48{,}62}{1862}\right)^2} - \frac{1}{2}\left(\frac{100-48{,}62}{4862}\right)$$

Executing operations:

$n = \sqrt{1.811 \div 4862 + \tfrac{1}{4}\,(.01056)^2} - \tfrac{1}{2}\,(.01056)$

$n = \sqrt{.000376 + .0000279} - .00528$

$n = \sqrt{.0004039} - .00528$

$n = .0201 - .00528 = .0148.$

In this case take $n = .015$.

By the aid of the tables prepared by the author which are embodied in this work this cumbersome operation could have been avoided in the following manner: Since r, v and s are given, C is obtained from the formula $v = C\sqrt{rs}$, from which we first obtain: $C = v \div \sqrt{rs}$. This operation executed in the case under consideration gives $C = 100$.

Now look through the different tables for the place when C is nearest to 100, when $\sqrt{r} = 1$, and for slope .0004. The table for $n = .013$ shows $C = 119.25$ and the table for $n = .015$ shows $C = 97.96$; thus it is seen that the coefficient of roughness should be .015. If absolute accuracy is wanted it may be found by interpolation; thus the coefficient n will be less than .015 and more than .013, in the same ratio as 100 lies between 97.96 and 119.25; difference between these two numbers is 21.29; hence as C decreases 21.29, n advanced from .013 to .015, or at the rate of .0001 per unit; as 97.96 is nearly equal to 98, multiply this by 2 and subtract from $.015 = .0148$, which is the number as found above.

Hence the determination of the coefficient n is quite readily done by the use of the tables with very little work.

ARTICLE XII.

EXPLANATION OF THE "C" TABLES.

This book contains 14 tables giving the factor *C* for 14 different degrees of roughness and for 29 different hydraulic radii, each for six different slopes. These were carefully selected for the purpose of providing sufficient data for all practical problems which might present themselves.

The author made the computations for the square root of the hydraulic radii ($\sqrt{r}$), which still further increases the range of the tables, so that they cover pipes from ½-inch diameter to conduits of 3,600 ft. diameter or rivers or canals of equivalent cross-section. The slopes shown vary from .001 to .000025, which covers practically all cases ever arising.

With these tables calculated ready for use, the solution of hydraulic problems becomes an easy matter, especially if use is made of the tables of hydraulic radii, wetted perimeters, areas of circular segments, and the many other original auxiliary tables, such as square roots, cube roots, etc.

A number of problems will be worked in succeeding articles which show the great adaptability of these tables and their consequent value as a time saver in reaching practical results. It is seen that the factor *C* grows as the hydraulic radius grows, and that the factor *C* is constant for all channels in which $\sqrt{r} = 1.8$, having the same coefficient of roughness n. It is also seen by examining the "*C*" tables that the factor *C* decreases with the slope for all channels in which $\sqrt{r}$ is less than 1.8, but the factor *C* increases as the slope decreases in all channels in which $\sqrt{r}$ is greater than 1.8.

ARTICLE XIII.

PROBLEMS. OPEN CHANNELS.

A. *Triangular Channel.*

1. *Find Velocity When the Other Quantities Are Given.* In Fig. 81 is shown a problem where a reservoir of water *A* is connected by a timber flume *C*, made of very smooth planed boards, to a reservoir *B* whose level is lower than *A* a distance of 3 feet. The length of flume is 6,600 ft., and cross section of flow is uniform throughout entire length; the surface levels in *A* and *B* are supposed to be constant.

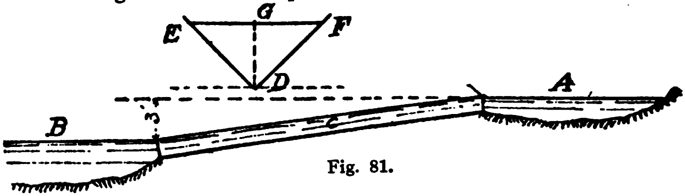

Fig. 81.

Find quantity of water discharging per second if the depth of flow *GD* equals 30 inches and angle *EDF* = 90°, the triangle *EDF* being the cross section of the flow.

Solution. Assume coefficient of roughness .011. Find the slope s by dividing 6,600 into 3, which gives .00045. To find r first find area of triangle *EFD*, which equals $GD \times EG = 2.5 \times 2.5 = 6.25$ sq. ft. The wetted perimeter equals $ED + DF$; $ED = \sqrt{12.50} = 3.535$; hence $r = 6.25 \div 7.07$.

$r = .8803$.

Take $\sqrt{r} = \sqrt{.8803} = .938$.

Consult "C" table under, $n = .011$ (Table III); in column $s = .0004$ we find $C =$ for $.9 = 133.79$.
for $1.0 = 138.92$.

Hence, for a one-tenth part of $\sqrt{r}$ C advances 5.13, or .0513 for 1/1,000 part of $\sqrt{r}$; so for .038 parts the advance is 38 times $.0513 = 1.95$; add this to 133.79 gives 135.74; this is the exact factor C for this case.

The mean velocity v is now determined from the formula: $v = C\sqrt{rs}$.

Substitute above values:

$v = 135.74 \times .938 \times .021.$

$v = 2.67$ ft. per second.

Hence quantity Q in cub. ft. per second.

$Q = 6.25 \times 2.67 = 16.687$ cub. ft.

2. *Find Size of Flume if the Other Quantities Are Given.* A variation of the problem may be worked by assuming

the slope as before, but require the size of flow for a required capacity.

Problem: Find depth GD to deliver a flow of 50 cub. ft. per second.

As stated before, $n = .011$ and $s = .00045$. Let $GD = x$; then x^2 = area of cross section of flow and $50 \div x^2 = v$.

Also $r = x \div 2\sqrt{2}$.

Substitute these values in $v = C\sqrt{rs}$, we get:

$50 \div x^2 = C\sqrt{xs \div 2\sqrt{2}}$

Square both sides:

$$\frac{2{,}500}{x^4} = \frac{C^2xs}{2.828}$$

$C^2x^5s = 2.828 \times 2{,}500.$

Assume C as above at 135 and $s = .00045$, we get:

$x = \sqrt[5]{2.828 \times 2{,}500 \div 135 \times 135 \times .00045}.$

$x = \sqrt[5]{7{,}070 \div 8.2} = \sqrt[5]{862.2}.$

The 5th root is best extracted by means of logarithms:

Log. 862.2 = 2.93561.

Divide by 5 = 0.58712.

Look up number corresponding to this logarithm:

$x = 3.865.$

Hence the depth of flow is 3.865, which will give the flume a capacity of 50 cub. ft. per second.

To check this back assume depth as 3.865 and ascertain flow: area = 3.865 × 3.865 = 14.938.

$r = 3.865 \div 2.828 = 1.367.$

$v = 135\sqrt{1.367 \times .00045}.$

$v = 135 \times .0248.$

$v = 3.348$ ft.

Multiply by area 14.938.

$Q = 50.6$ cubic ft; thus it is seen that the problem checks back correctly.

B. *The Rectangular Channel.*

3. *Problem.*

A flume of rectangular cross section (see Fig. 82) 24x24" is tapping a river at its upper end and is 3 miles long, having a uniform grade of 5 ft. per mile. Find velocity and quantity of water if factor $n = .012$ and if depth of flow = 16".

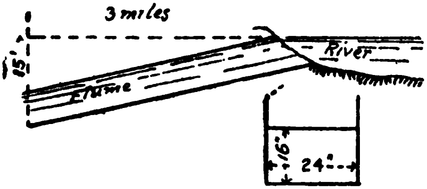

Fig. 82.

Solution. In

this case n, r and s are given and the problem can be worked accordingly.

Area of flow $= 2 \times 1\frac{1}{3} = 2\frac{2}{3} = 2.67$ sq. ft.

Wetted perimeter $= 2 + 1\frac{1}{3} + 1\frac{1}{3} = 4\frac{2}{3} = 4.67$ ft.

Hydraulic radius $= 2.67 \div 4.67 = .5717$.

$\sqrt{.5717} = .756$.

$s = 5 \div 5{,}280 = .00095$.

Consult table No. IV ($n = .012$) and factor C will be found under slope .001 (which is nearest to .00095) and opposite $\sqrt{r}$ column, between .5 and .6, or the proportional part for .5717; it will be near enough to compute C for .57;

C for $.50 = 94.52$.

C for $.60 = 103.44$.

Difference: 9.12

This is a variation of .91 for .01 increase in $\sqrt{r}$; thus for .07 it is 7 times $.91 = 6.37$; add this to 94.52 gives 100.89, as C for $\sqrt{r} = .57$.

In the fundamental formula:

$v = C\sqrt{rs}$. Substitute values:

$v = 100.89 \times .57 \times \sqrt{.00095}$.

Consult table XIV of square roots; under x find $\sqrt{.0009} = .03$.

Under $\sqrt{.0010}$ you find .03162.

Difference: .00162.

Hence $\sqrt{.00095}$ equals .03 plus half of .00162, or $\sqrt{.00095} = .03081$.

$v = 100.89 \times .57 \times .03081$.

$v = 1.7678$ ft. per second.

The quantity of water per second equals cross section times mean velocity:

$Q = 2.67 \times 1.77 = 4.726$ cub. ft. per sec.

C. *Trapezoidal Channel.*

4. *Problem.*

A mill race of trapezoidal cross section has a bottom width of 3′ 4″ and the depth of flow is 2 ft.; the slope is 6 inches in 1,000 ft.; the side walls of the channel have an inclination of 60° to the horizon, and bottom and walls are constructed of concrete plastered fairly smooth. Find mean velocity of flow.

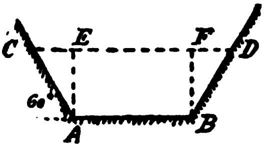

Fig. 83.

Solution. Let Fig. 83 represent the cross section of channel, and AE be depth of flow $= 2$ ft.; then $AC = BD = 2 \div \sin 60°$, and $CE = FD = 2 \div \cos 60°$; this makes $AC = 2.309$ ft. and

$CE = 1.1545$ ft. The area of flow and length of wetted perimeter can now be readily determined: Area of flow: $(AB + CD) \div 2 \times AE = (3.33 + 1.13 + 3.33 + 1.13) \div 2 \times 2 = 8.91$ sq. ft.; the wetted perimeter $= CA + AB + BD = 2.309 + 3.333 + 2.309 = 7.951$ ft.

Hence hydraulic radius $= 8.91 : 7.951 = 1.1205$.

And $\sqrt{1.1205} = 1.059$.

If we now assume the coefficient $n = .013$ we have all the necessary elements to make use of the tables $s = 5 \div 1{,}000 = .0005$: table No. V ($n = .013$), under slope .0004 (which is nearest to our case), we find C opposite $\sqrt{r}$; for

$\sqrt{r} = 1.0 \; C = 115.29$; for

$\sqrt{r} = 1.2 \; C = 123.06$; difference for $.2 = 7.77$, then difference for $.001 = .0388$, and for $.044$ it will be $44 \times .0388 = 1.7072 = 1.71$; add this to 115.29 gives 117.00 for factor C in this case.

Substitute the known quantities in formula: $v = C \sqrt{rs}$

$v = 117.0 \times 1.059 \times .022 = 2.726$ ft.

The quantity of flow per second $= av = 8.66 \times 2.726 = 23.607$ cub. ft. per second.

5. *The Chicago Drainage Canal.*

This canal connects Lake Michigan with the Mississippi river via the Chicago, Desplaines and Illinois rivers, and is a splendid example of a successful solution of a great sanitary and engineering problem, providing at once means for diverting Chicago's great Sewer, the Chicago river, from the same city's great reservoir of drinking water, Lake Michigan, and at the same time introducing sufficient dilution to prevent evil effects in its course to the people in the Illinois and Mississippi river valleys; this on the one hand prevents further pollution of Chicago's water supply and also forms the first and most costly link in a deep water way from the Great Lakes to the Gulf.

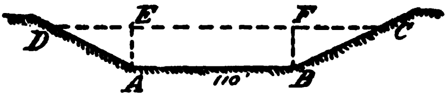

Fig. 84.

The canal proper starts at Robey street, Chicago, and extends to Summit, Ill., a distance of 7.8 miles, having a cross section shown in Fig. 84, 110 ft. wide at the bottom, and side slopes 2 to 1; the elevation of the bottom at this point is —24.448 (which means 24.448 feet below lake level). From Summit to Willow Springs, a distance of 5.3 miles, the cross section is indicated in Fig. 85, being 202 ft. wide on the bottom with side slopes 2 to 1. From Willow Springs to Lockport, 14.95 miles, the channel is cut in rock, with a width on the bottom of 160 feet, with sides nearly perpendicular (162 wide 22 ft. above bottom). This section is shown in Fig. 86. The grade through the earth section is 1 in 40,000 ft., or $s = .000025$; in rock the grade is 1 in 20,000, or $s = .00005$.

At Lockport, where the waters of Lake Michigan join those of the Desplaines river, ample controlling works have been erected to regulate the flow to necessary requirements. While at present the flow is about 400,000 cub. ft. per min., it is expected that this will be gradually increased so that when the population of Chicago becomes 3,000,000, the flow of lake water will reach 600,000 cub. ft. per min., or 10,000 cub. ft. per second. Before this latter volume is sent down the canal the earth portion from Robey street to Summit will have to be widened out to 202 ft. on the bottom, conforming to section shown in Fig. 85. The Chicago river connects the canal with Lake Michigan and has a length of 6 miles from Robey street to Lake street. Its width was quite variable at the time the drainage canal was constructed, but has been improved, and has now almost a uniform width of 200 feet, a depth of 26 ft. for 100 feet in midchannel, and 16 feet on the dock lines, as shown in Fig. 87.

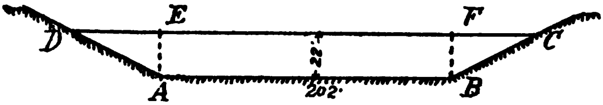

Fig. 85.

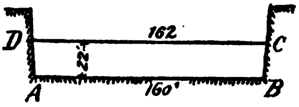

Fig. 86.

We can now consider several of the problems which present themselves in connection with the drainage canal.

6. *Problem.*

A flow of 300,000 cub. ft. per minute stands 22 ft. deep in the narrow channel shown in Fig. 84. What is the coefficient of roughness n?

The cross sectional area of flow is easily computed and is 3,388 sq. ft.; the flow is 5,000 cubic ft. per second, hence the mean velocity $v = 5{,}000 \div 3{,}388 = 1.48$ ft. per second.

The hydraulic radius is next computed:

$r =$ area $DABC \div DA + AB + BC.$

$r = 3{,}388 \div (49.19 + 110 + 49.19).$

$r = 3{,}388 \div 208.38 = 16.26$.

Then $\sqrt{r} = 4.032$.

The slope $s = .000025$ and $\sqrt{.000025} = .005$.

The data on hand will give now the factor C since $C = v \div \sqrt{rs}$, hence:

$C = 1.48 \div 4.032 \times .005$.

$C = 1.48 \div .02016 = 73.41$.

Consult table No. 12, ($n = .040$) and we find opposite 4.0 in slope column .000025 the factor C 78.45. Next consult table No. XIII ($n = .050$), we find opposite 4.0 in slope column .000025 the factor $C = 65.02$, hence the value of n in this case is greater than .040, and less than .050; by interpolation we find the difference is 13.43, that is, the factor C decreases 13.43 while the factor n increases from .040 to .050, or at the rate of 1.343 for each .001; the factor C in question equals 73.41, or 5.04 smaller than the C belonging to .040; hence as often as 1.343 is contained in 5.04, that many times will .001 have to be added to .040 for the required factor n; this gives 3.7, or (nearest) 4 times .001 added to .040 gives .044 as factor n for this section.

7. *Problem. Find the slope s.*

As this volume of the water strikes the earth channel between Summit and Willow Springs where the cross section is considerably larger, it stands to reason that, all other things being equal, there will be a reduction in the slope s, on account of the reduced velocity. The area in Fig. 85 equals 5,412 sq. ft., and the wetted perimeter equals 300.38 ft., hence:

$r = 5{,}412 \div 300.38 = 18.017$.

Then $\sqrt{r} = 4.25$.

The velocity $v = 5{,}000 \div 5{,}412 = .924$ ft.

From $v = C\sqrt{rs}$ we have by transposition:

$s = v^2 \div C^2 r$, in which equation v and r are known. To find C we assume that the factor n is the same as in the preceding case, namely, .044, and again find C by interpolation, as follows: If n were $= .040$, C is found from table No. XII (last column), between $\sqrt{r} = 4.0$ and 5.0; C for $4.0 = 78.45$; for $5.0 = 89.28$; difference $= 10.83$; hence difference for $.25 = 2.71$; add this to 78.45 gives 81.16 as C for $n = .040$; proceed likewise for factor C if $n = .050$, which produces 2.45 to be added to $65.02 = 67.47$ Now we have C for $.040 = 81.16$ and C for $.050 = 67.47$; hence difference 13.69 or 1.37 for .001; multiply by 4 gives 5.48 to be subtracted from 81.16 (because C diminishes as n increases) gives 75.68 as factor in C for the case under consideration.

If we substitute now the known values in:

$s = v^2 \div C^2 r$, we obtain:

$s = .924^2 \div 75.68^2 \times 18.017$.

$s = .000008$.

8. *Problem.*

We will next consider the flow of 5,000 cub. ft. of water through the rock section shown in Fig. 86. The cross section of the flow equals 3,542 sq. ft. and the velocity 1.41. As this section eventually will have to accommodate a flow of 10,000 cub. ft. per second, the slope of the flow in this case will be only half the final slope of .00005, or $s = .000025$, and we can now determine the factor n for the rock cut.

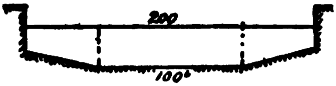

Fig. 87.

Area of flow = 3,542 sq. ft.

Wetted perimeter = 160 + 22.02 + 22.02 = 204.04.

$r = 3{,}542 \div 204.04 = 17.3596$.

$\sqrt{r} = 4.167$.

The factor C can now be determined since

$C = v \div \sqrt{rs}$, namely:

$C = 1.41 \div 4.167 \times .005 = 67.1$.

To find coefficient of roughness n proceed as follows: In table No. XII (where $n = .040$), opposite $\sqrt{r} = 4$, we find under $s = .000025$, $C = 78.45$. In table No. XIII, where $n = .050$, we find 65.02, and since our factor $C = 67.1$ lies between the two, it follows that coefficient n is greater than .040 and smaller than .050, and found exactly by interpolation, as explained above; this brings $n = .048$.

Right here it may be said that these values of n are taken entirely too high; the designers of the drainage canal evidently desired to be on the safe side when they computed cross sections and slopes, for it stands to reason that a channel of such regular section should have a much lower factor of roughness than what is indicated above; they should not be much more than .025. However, it is well that the capacity is above present requirements; there are splendid controlling works at Lockport which regulate the flow precisely to what is wanted by increasing or decreasing the slope of the flow.

The applications shown above relate merely to the principles involved; as a matter of fact the slopes in use are much smaller than what is indicated and they are determined by the controlling works at Lockport.

9. *The Chicago River.*

The foregoing subject is closely related to the Chicago river, which will be discussed in this paragraph from the hydraulic point of view. The topography of the water shed of the Chicago river near Lake Michigan has a very gentle slope toward that body of water, and the different branches of the river are therefore particularly adapted for harbor purposes. Before the completion of the main drainage canal, when the sewage of Chicago was pumped from the Chicago river into the Illinois and Michigan canal, it was a common occurrence to notice the river flow first toward the lake and then toward the canal; the former occurred after heavy rain storms when the pumps at Bridgeport were unable to cope with the flood. At such times and during protracted floods the black waters of the river would enter the lake and often reach the intake cribs of the Chicago water supply, contaminating same. The completion of the sanitary canal gave Chicago permanent relief from this, and gave a positive outlet to the Chicago river into the drainage basin of the Desplaines river.

The Chicago river within the limits of Chicago is principally used as a harbor and for navigation; this implies that the velocity of the current should be low at all times to prevent accident to shipping. For this reason the cross section of the river is made ample.

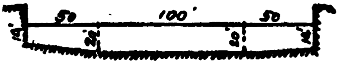

Fig. 88.

Figure 88 shows an older typical section of the Chicago river, having a width of 200 ft. and a depth varying from 14 ft. at the dock lines to 20 ft in midriver. The effective flow area is composed of $16 \times 50 = 850 + 100 \times 20 = 2{,}000 + 16 \times 50 = 850 = 3{,}700$ sq. ft.; the wetted perimeter equals: $14 + 50.36 + 100 + 50.36 + 14 = 228.72$, hence the hydraulic radius equals:

$$3{,}700 \div 228.72 = 16.18, \text{ hence:}$$

$$\sqrt{16.18} = 4.02.$$

The maximum quantity of water flowing through equals 5,000 cub. ft. per second, hence:

$$v = 5{,}000 \div 3{,}700 = 1.35 \text{ ft. per second.}$$

This appears a very moderate velocity, and so it is if the above indicated cross section was fully obtained everywhere. However, in many places the effective cross section is taken up by bridge piers and abutments, vessels anchored and vessels moving up and down, so that in many places this cross sections appears cut down one-half, which naturally doubles the velocity of the flow. Again it must be remembered that the fast increasing population of Chicago demands an increased flow of lake water through the drainage canal, so that it may be assumed that the 3,000,000 population for Chicago will be obtained within the next ten years, the flow per second through the river will have increased to 10,000 cub. ft. per second, and the velocity to 2.7 ft. per second; as this is rather high for the safety of shipping, steps have been taken to increase the depth of the river ultimately to 26 feet, which will increase the hydraulic efficiency of the river very much. The north branch of the Chicago river will be diluted by an influx of 1,000 cub. ft. per second of lake water through the Evanston ditch, now being constructed, 583 cub. ft. per second through the Lawrence avenue intercepting sewer tunnel, and about 300 cub. ft. per second through the Fullerton avenue conduit. Also the 20 ft. diameter tunnel in Thirty-ninth street will deliver through the south arm of the Chicago river 2,000 cub. ft. of lake water per second, so that these various auxiliary channels supply 3,883 cub. ft. of lake water per second, leaving less than 7,000 cub. ft. as a maximum to be supplied through the main river.

ARTICLE XIV
CLOSED CHANNELS

1. *Rectilinear Channels.*

Of the various forms of closed channels the cylindrical and composite forms will be especially considered, as all the others bounded by right lines are readily solved by the preceding articles; for instance a closed channel of rectangular section will be considered an open channel until it flows full; then the hydraulic radius undergoes an immediate change. As shown before, the hydraulic radius varies directly with the height in an open channel; but if *ABCD* (Fig. 89) represents a closed channel, then when the height of flow reaches the top *DC*, the wetted perimeter is increased to the extent of the face *DC*, and causes a considerable reduction in the hydraulic radius; for instance, let $AB = 4$ ft. and $AD = 3$ ft; then just before the water reaches *DC* we have

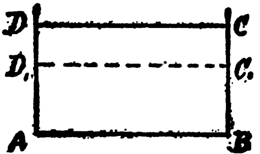

Fig. 89.

$r = 3 \times 4 \div 3 + 4 + 3 = 12 \div 10 = 1.2.$

Then the moment the water reaches *DC*:

$r = 3 \times 4 \div 3 + 4 + 3 + 4 = 12 \div 14 = .857;$ in case the cross section is a square with the side a, then $r = a^2 \div 4a = a/4$ in the case of a closed channel.

Similar deductions may be made in regard to closed channels of triangular and trapezoid sections, each of which is considered an open channel until the roof becomes immersed, when the reduction of r must be considered.

2. *Cylindrical Channels.*

Channels of circular cross sections are the most generally used and of the greatest industrial importance, as most sewers and water tunnels are at present of that form, and practically all water mains and supply pipes. While most sewers are ordinarily flowing only partially full and only occasionally are completely filled, water pipes as a usual thing are flowing not only full but often under considerable pressure; therefore the problems of the cylindrical channel comprise three varities: First, channels flowing partially full; second, those flowing full, and third, those flowing full under a considerable pressure. These various phases are now explained by different applied problems.

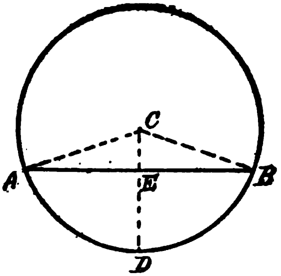

Fig. 90.

3. *Applied Problem.*

A brick sewer 16 ft. diameter drains a territory of 4,000 acres, and the dry weather flow delivers .011 cu. ft. per second per acre, while the storm flow equals .10 cu. ft. per second per acre; the slope of sewer is .0001 and coefficient of roughness .015; find depth and velocity of dry weather flow, also depth and velocity of storm flow.

Solve dry weather flow first. Total flow equals 4,000 × .011 = 44.0 cub. ft. per second. To begin, assume a probable hydraulic radius, say 1.44 feet. Draw a diagram as indicated in Fig. 90, in which the line *AB* represents the height of flow and *ED* is the versed line.

Then $\sqrt{1.44} = 1.2$; turn to table VI; we find under slope .0001 opposite $\sqrt{r} = 1.2$, factor $C = 101.77$, say 102 for short; then $v = 102 \times 1.2 \times .01 = 1.124$ ft. per second; find area corresponding to $R = 1.44$, in table XV. Divide 1.44 by 16 = .09, which is hydraulic radius constant and agrees with area-constant .071; multiply .071 by the square of diameter 16 × 16 = 18.18, which is area of segment; multiply by velocity 1.124, gives the quantity of flow = 20.36 cub. ft., which is too small, and hence the value of *r* must be increased.

Next assume $r = 1.96$, then $\sqrt{r} = 1.4$.

In table VI under slope .0001 opposite $\sqrt{r} = 1.4$ we find factor $C = 108.96$, say 109.

Then $v = 109 \times 1.4 \times .01 = 1.526$.

Find area corresponding to $R = 1.96$ (table XV). Divide 16 into 1.96 = .125, which is hydraulic radius constant in table XV, lying about half way between .1206 and .1311; so the corresponding area constant lies between .11182 and .12811, or .1199; multiply by $16^2 = 256$ which gives 30.48; multiply this by $v = 1.526$, gives 46.52, cub. ft., which is slightly too great, but near enough for practical purposes; hence the versed sine of flow will be between .20 and .22, say .21, which, multiplied by 16, gives 3.36 ft. for depth of dry weather flow.

To find the depth and velocity of the storm flow, proceed in like manner. Calculate the maximum storm flow for 4,000 acres at .10 = 400 cub. ft. water. Assume a probable hydraulic radius at 4.0, or sewer flowing half full; then $\sqrt{4.0} = 2.0$. Turn to table VI; we find under slope

.0001 opposite $\sqrt{r} = 2.0$ the factor $C = 125.13$; then $v = 125. \times 2 \times .01 = 2.5$; find area corresponding to $R = 4.0$, which is half of area of full 16 ft. circle = 100.53; multiply this by 2.5 gives 251.32 cub. ft. per second; this is too small, so assume a larger hydraulic radius; assume $\sqrt{r}$ at 2.20; table VI shows $C = 125.13$, say 125. at $\sqrt{r} = 2.0$; at $\sqrt{r} = 2.5$; $C = 134$, so for $\sqrt{r} = 2.2$ the factor C equals $2/5 \times 9 = 3\text{—}3/5$, which, added to 125.13, makes 128.73, or about 129.0.

Then $v = 129 \times 2.2 \times .01 = 2.84$ ft. per second. Find area corresponding to $R = 4.84$; divide 16 into 4.84 = .302 (hydraulic radius constant, table XV); this agrees with area constant .63185 and versed sine .75; so multiply .63 by $16^2 = 161.28$ sq. ft.; multiply this with $v = 2.84$, gives 458.03 cub. ft. per second; this indicates that the maximum flow of 400 cub. ft. per second will not reach the $.75 \times 16 = 12$ ft. and r is taken too high, but we have limits established, as the depth of flow lies between 8 ft. and 12 ft., and $\sqrt{r}$ must lie between 2.0 and 2.2; assume $\sqrt{r} = 2.1$ then $C = 127$, and $v = 127 \times 2.1 \times .01 = 2.67$ ft., $r = 4.41$; divide 16 into 4.41, gives .28; in table XV this brings area constant .492 and versed since .60; multiply .49 by $16^2 = 145.44$ sq. ft.; multiply by 2.67, gives 388.32 cub. ft. This is very near the requirement, and by multiplying 16 by .6 we get depth of flow, 9.6 ft. This evidently shows that the size of the 16 ft. sewer is too large for the maximum flow of 400 cub. ft. per second.

4. *Find the Necessary Size.*

Another useful problem to work would be: Find size of a circular sewer to carry a maximum flow of 400 cub. ft. per second when $s = .0001$ and $n = .015$. As the maximum flow obtains if height of flow area = .930 we will assume the sewer flowing .93 full at that rate; the hydraulic radius constant is then .2918, so that in a circle of diameter = 1, the area would be .7612, and $r = .292$, and wetted perimeter 2.61 (see table XV). Assume a sewer 12 ft. in diameter; then at $.93d$ area equals $12 \times 12 \times .7612 = 109.61$ sq. ft.; divide this into 400, gives $v = 3.65$. Now check v as follows: $r = 12 \times .292 = 3.50$ and $\sqrt{r} = 1.86$. In table VI find $C = 121$; then $v = 121 \times 1.86 \times .01 = 2.25$. It is seen that v is too small, hence a larger sewer must be used.

Try $d = 14$ ft.; then at $.93d$ area equals $14 \times 14 \times .76 = 148.96$ sq. ft.; divide this into 400 gives $v = 2.77$ ft. per second. Now $r = 14 \times .292 = 4.09$, and $\sqrt{r} = 2.02$;

in table VI we find C for $\sqrt{r} = 2.02 = 125$; then $v = 125 \times 2.02 \times .01 = 2.53$ ft. per second, giving a total flow of 348.56 cubic feet.

This diameter is still too small, so try $d = 15$ ft.; flow area at $.93d = 15 \times 15 \times .76 = 171.0$ sq. ft.; divide into 400 gives $v = 2.34$, $r = 15 \times .292 = 4.38$ and $\sqrt{r} = 2.09$; $C = 127$ (table VI); then: $v = 127 \times 2.09 \times .01 = 2.63$, which is greater than necessary, as $2.63 \times 171 = 449.73$ cub. ft.

This size would probably be selected, as there would be an excess in capacity of nearly 50 cub. ft. per second, which would be on the safe side.

5. *Find the necessary slope.*

Another class of problems will present themselves. Suppose it should be desirable, for good reasons, that the diameter of the sewer should not exceed 12 ft.; then it would be necessary to increase the slope s, which factor is now to be developed.

When flowing .93 full the flow area = 109.61, requiring a velocity of 3.65 ft. per second (see above). On a slope $s = .0001$ we obtain a velocity of 2.25; let x = required slope, then:

$$\sqrt{x} : \sqrt{.0001} = 3.65 : 2.25, \text{ hence}$$

$$\sqrt{x} = \frac{.01 \times 3.65}{2.25} = .0162; \text{ then } x = .0162 \times .0162;$$

hence $s = .000266$, which is the required grade. To check this, find flow of 12 ft. sewer flowing .93 full on above grade. R is found from table XV by multiplying .2918 by $12 = 3.502$, and $\sqrt{R} = 1.87$. From table VI under slope .0002 and for $\sqrt{r} = 1.8$ we find $C = 120.5$; hence we find $v = 120.5 \times 1.87 \times .0162 = 3.65$ ft.; the flow area in the 12 ft. sewer equals $12 \times 12 \times .7612 = 109.61$ sq. ft., hence the maximum flow will equal $109.61 \times 3.65 = 400$ cub. ft. per second. This is slightly less than the real capacity, as the factor C was evidently taken for $\sqrt{r} = 1.8$; for $\sqrt{r} = 1.87$ it should be taken larger in proportion, as follows:

C for $1.8 = 120.5$.
C for $2.0 = 124.5$.

This is a variation of 4 units for an advance of .2 in the $\sqrt{r}$; then the proportional part for 1/10 would be 2 units, for 1/100 it would be .2, and for 7/100 it would be 1.4; this should be added to 120.5, which makes the true

$C = 121.9$; this increases $v = 3.79$ and the maximum capacity to 415.52 cub. ft. per second.

The foregoing illustrations show how to handle problems of circular conduits flowing partially full.

6. *The Composite Section.*

To compare the composite section with the true circular one, let us solve the last problem for the composite section.

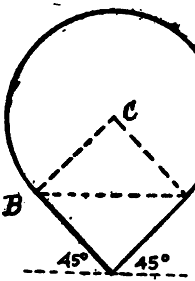

Fig. 91.

Find maximum capacity of composite channel having a diameter of 12 ft., using $S = .000266$ and $n = .015$.

Draw diagram (Fig. 91) showing outline of section.

The maximum capacity of the composite section is when flowing .92 full of circular depth. From table XVI we find opposite .92 circular depth the constant for $r = .2908$ and for area $= .8096$; hence $r = 12 \times .2908 = 3.49$. $\sqrt{r} = 1.87$, and flow area $= .8096 \times 12 \times 12 = 116.58$ sq. ft.; from table VI, under slope .0002, we find C opposite $\sqrt{r} = 1.8 = 120.50$, and with the adjustment noted in the preceding paragraph gives us $C = 121.9$, hence:

$v = 121.9 \times 1.87 \times .0162 = 3.79$ sq. ft. per second.

The total capacity then will be:

$116.58 \times 3.79 = 444.84$ cu. ft. per second.

It may be noted that the velocity is the same in the 12 ft. circular and 12 ft. composite section, but there is a material gain in capacity, due to the greater flow area.

Another advantage of the composite section lies in the fact that for dry weather flow it gives much more satisfactory velocities than the circular section; this will well be illustrated in one of the following applications:

7. *Find flow area and velocity.*

Suppose the 12 ft. composite sewer is to take care of dry weather sewage flow of 40 cu. ft. per second; find flow area and velocity, assuming same slope as above.

From table XVI select a random depth, say .3, which would mean a depth of $12 \times .3 = 3.6$ ft; the area constant opposite $.3 = .090$, and hydraulic radius constant $= .108$; hence flow area $= .09 \times 144 = 12.96$ sq. ft., and hydraulic radius $= .108 \times 12 = 1.296$, and $\sqrt{r} = 1.14$.

Refer to table VI under slope .0002 and opposite $\sqrt{r} = 1.0$, we have $C = 96.11$; opposite $\sqrt{r} = 1.2$ we have $C=$

103.98; this shows a difference of 7.87 for .2, or .3935 for 1/100; hence for .14 we should add 14 × .3935 = 5.51 to 96.11, which gives $C = 101.62$; hence $v = 101.62 \times 1.14 \times .0162 = 1.88$ ft. per second. Multiply this with 12.96 gives = 24.37 cu. ft. flow, which is not enough, as we require a flow of 40 cu. ft. per second.

Next try a depth of .407; from table XVI we find area constant .1654, and r constant = .1449; multiply this by 12, we get:

$R = 12 \times .1449 = 1.74$ and $\sqrt{R} = 1.32$.

Area = 12 × 12 × .1654 = 23.82 sq. ft.

Refer to table VI. Under slope .0002 opposite $\sqrt{R} = 1.2$ we find $C = 103.98$.

Opposite $\sqrt{R} = 1.4$ $C = 110.51$.

Difference = 6.53 for .2; for 1/100 .3265, hence for .12 we must add 12 × .3265 = 3.92 = 107.90.

Now $v = 107.9 \times 1.32 \times .0162 = 2.306$ ft. per second.

Multiply this by 23.82 gives 54.92 cu. ft. per second.

This is too much, but we established limits for the flow height.

Next try in table XVI a depth of .387; the area constant is .1497.

R constant = .13723; multiply by 12 gives $R = 1.65$ and $\sqrt{R} = 1.28$. Area = .1497 × 12 × 12 = 21.557 sq. ft.

From table VI find C for $\sqrt{R} = 1.28$, as explained above, gives us $C = 106.62$.

$v = 106.62 \times 1.28 \times .0162 = 2.21$ ft. per second.

Multiply 2.21 by 21.557 = 47.62 cu. ft. per second.

This is still too large, but gives a very good idea about the depth of flow; for accurate work we will try another depth.

Take .367 in table XVI; area constant = .1347, R constant = .1298; multiply by 12 gives $R = 1.56$; $\sqrt{R} = 1.25$; area = .1347 × 12 × 12 = 19.397 sq. ft.

From table VI find C for $\sqrt{R} = 1.25 = 105.61$.

Then $v = 105.61 \times 1.25 \times .0162 = 2.14$ ft. per second.

Multiply 19.397 by 2.14 = gives 41.51 cu. ft per second.

This Answer fulfills the condition; so the Composite Section has a flow depth of .367 × 12 ft. = 4.4 ft., and a velocity of 2.14 ft. per second; which is sufficient to prevent sedimentation.

8. *Comparisons between Circular and Composite Sections:*

For very small quantities the composite section gives better velocities than the circular; to illustrate, find velocity and flow area for a flow of 8 cu. ft. per second in a conduit 12 ft. in diameter. For the composite section, assume depth .18; then area constant = .0324 and R constant .0648. Then $R = .78$ and $\sqrt{R} = .88$; the flow area = 4.67 sq. ft.

From table VI find C for $\sqrt{R} = .88 = 90.35$ and $v = 90.35 \times .88 \times .0162 = 1.29$ ft. per second.

Multiply 4.67 by 1.29 = 6.02 cu. ft. per second, which is not enough. Assume depth .2, area constant .04, R constant .072; then $R = .864$ and $\sqrt{R} = .93$, and flow area 5.76.

Find C in table VI for $\sqrt{R} = .93 = 92.83$, and $v = 92.83 \times .93 \times .0162 = 1.40$ ft. per second.

Multiply 5.76 by 1.40 = 8.06 cu. ft. per second, which just covers the case, giving a depth of flow of 2.4 ft. and a flow velocity of 1.4 ft. per second.

To find the depth of flow and velocity in the circular sewer of 12 ft. diameter, we may assume that the velocity will not exceed 1 ft. per second—then the flow area will be 8 sq. ft. Since the area of a segment equals d^2 times area constant, the area constant equals area divided by d^2; hence divide 144 into 8, which gives area constant = .055; in table XV we find area constant = .05338 and R constant = .0754; then $R = .905$ and $\sqrt{R} = .951$; area of segment = .05338 × 144 = 7.69 sq. ft.

Then in table VI find C under slope .0002 = 91.53 for .90 for $\sqrt{R} = 1.0$; $C = 96.11$, so C for .95 will be half way between 91.53 and 96.11 or 93.82.

Then $v = 93.82 \times .95 \times .0162 = 1.44$ ft per second. Multiply this with 7.69 gives 11.07 cu. ft. per second, which is too much.

The above trial corresponded to a versed sine of .12 let us try .09; then the area constant = .03501 and R constant = ..0574, hence, $R = .69$, $\sqrt{R} = .83$, and area of segment 5.04 sq. ft.

In table VI we find C for $\sqrt{R} = .83 = 87.9$, and $v = 87.9 \times .83 \times .0162 = 1.18$ ft. per second. Multiply 504 by 1.18 = 5.95 cu. ft. per second. This is not enough. Let us next try versed sine .1; the area constant is .04087, R constant .0635, hence $R = .762$ and $\sqrt{R} = .873$, and area of segment 5.885 sq. ft.

The factor C for $\sqrt{R} = .87 = 89$.

$v = 89.0 \times .87 \times .0162 = 1.25$.

Multiply 5.89 by 1.25 gives 7.36 cu. ft. per second.

This is not quite enough, but shows that the versed sine is slightly greater, but, we know that the versed sine of .12 is too great, and the real one is a very small fraction over .1, perhaps .105.

The solution shows, however, that while we have a velocity of 1.40 ft. per second in the composite section, we have only a velocity of slightly more than 1.25 ft. per second in the circular conduit.

ARTICLE XV

PIPES FLOWING FULL UNDER PRESSURE

1. *General Deductions.*

Whenever flow of water in a pipe is produced by pressure the head equivalent to such pressure can be readily computed. For instance, where the pressure is given as 20 lbs. per square inch it is equivalent to the height of a column of water producing 20 lbs. pressure per square inch. It requires a column of water 2.309 feet high with a cross-section of 1 sq. in. of water to produce a pressure of 1 lb. on its base; hence 20 lbs. pressure would be equivalent to a water column of 20 × 2.309 = 46.18 ft.; if such a pipe would have a length of 3,000 ft. then the head would be 46.18 ft. in 3,000 ft. and the factor $s = 46.18 \div 3{,}000 = .01539$.

In general let $l =$ length of pipe,
$p =$ pressure in pounds per square inch,
$h =$ head in feet,
$d =$ diameter of pipe and
$s =$ sine of slope,
then $s = h \div l$,
$h = 2.31p$,
hence $s = 2.31p \div l$.

As the pipes are constantly filled, $R = d/4$. If we transform the fundamental formula

$$v = C\sqrt{Rs}$$

by substituting above values for R and s we obtain:

$$v = C\sqrt{d/4 \times 2.31p \div l}.$$

$$v = C\sqrt{2.31pd \div 4l}.$$

Simplifying we obtain:

$$v = \frac{C}{2}\sqrt{2.31pd \div l}. \qquad (1)$$

From this formula we can develop any one factor when all the others are known, thus we can find the pressure p necessary to produce a certain velocity:

$$p = 4lv^2 \div 2.3dC^2; \qquad (2)$$

also find the diameter of a pipe to produce a given velocity under a given pressure and for a given length of pipe:

$$d = 4\,lv^2 \div 2.3C^2p; \qquad (3)$$

also find the length l when all the other factors are given:

$$l = 2.3\,C^2dp \div 4v^2, \qquad (4)$$

and lastly the constant C may be determined if all other factors are known:

$$C = \sqrt{4lv^2 \div 2.3pd}. \qquad (5)$$

These formulas are very simple and extremely useful in determining the different requirements of hydraulic problems.

Since $h = 2.31p$, then $p = \frac{h}{2.31}$ or $p = .433h$.

This shows that the head under which a certain pipe is flowing can be reduced to pressure in pounds per square inch by multiplying the head in feet by the constant .433.

Thus the pressure in a pipe, in which the water is flowing under a head of 10 ft. would amount to $10 \times .433 = 4.33$ lbs. per square inch.

2. *Find Velocity in Cast-Iron Pipe.*

The pressure in a system of water works is 25 lbs. per square inch; what will be the velocity in a 24-in. cast-iron pipe 5,000 ft. long, using the coefficient of roughness .011?

Solution—First find the constant C; the hydraulic radius is $2 \div 4 = \frac{1}{2}$ and $\sqrt{r} = .707$, or say .71. In Table III in $\sqrt{r}$ column opposite .7 factor $C = 123.11$; opposite .8 factor $C = 129.77$; difference 6.66; hence add .67 to $123.11 = 123.78$; being factor C for $\sqrt{r} = .71$. Next substitute values in formula: $v = \frac{C}{2} \sqrt{2.3pd \div l}$;

$$v = 123.78 \div 2 \sqrt{2.3 \times 25 \times 2 \div 5{,}000};$$
$$v = 61.89 \sqrt{.023};$$
$$v = 61.89 \times .151 = 9.45 \text{ ft. per second.}$$

3. *Find Pressure in Cast Iron Pipe.*

Problem—What pressure is necessary to produce a velocity of 6 ft. per second in a 36-in. cast-iron pipe 1 mile long, using factor $n = .012$?

Solution—Again find factor C first; $r = 3 \div 4 = .75$ and $\sqrt{.75} = .866$, say .87; in Table IV in $\sqrt{r}$ column we find opposite .8, $C = 117.21$; opposite .9, $C = 122.68$; difference, 5.47; hence add $7 \times .547 = 3.83$ to 117.21, making $C = 121.04$.

Next apply formula No. 2 in Article xv, substituting given values:

$$p = 4 \times 5280 \times 6 \times 6 \div 2.3 \times 3 \times 121.04 \times 121.04.$$
$$p = 760320 \div 101.119 = 7.52 \text{ lbs. per square inch.}$$

4. *Find Diameter of Cast-Iron Pipe.*

Problem—It is proposed to supply water to a cast-iron main under a pressure of 12 lbs. per square inch. What must the diameter be of the pipe if the length of pipe 6,200 ft., the required velocity is 5 ft. per second and factor $n = 012$?

Solution—The selection of the factor C will be difficult, as the diameter of the pipe is not known and a trial factor C must be used.

In formula 3 in Article xv assume $C = 100$—then we have:

$d = 4 \times 6200 \times 5 \times 5 \div 2.3 \times 12 \times 100 \times 100.$

$d = 620000 \div 276000 = 2.246$ ft.

Next we must check back and see what the true factor C is for $d = 2.246$; as follows:

$r = .561$; $\sqrt{r} = .75$; looking in Table IV the factor C for $\sqrt{r} = .7 = 110.88$ and factor C for $\sqrt{r} = .8 = 117.21$, hence factor C for $\sqrt{r} = .75 = 114.04$.

If we now use 114 as the factor C in the above composition we obtain:

$d = 620000 \div 358690 = 1.73.$

This shows that factor C is taken too high and that 1.73 is too small. Adding now 1.73 and 2.25 gives 3.98 and divide by 2 as a mean gives very nearly a diameter of 2 ft., which must be again checked up by the true factor C; in Table IV find C for $\sqrt{r} = .71$; for .7, $C = 110.88$; for .8, $C = 117.21$; difference 6.33, hence add .63 to 110.88, making $C = 111.51$.

Now try formula 3 again, using 111 as factor C:

$d = 620000 \div 340060 = 1.82$ ft.

For general work a 2 ft. diameter pipe would be the practical solution of the problem, though a 22-inch diameter pipe (if the exact size must be had) would undoubtedly be found sufficient; to try this bear in mind commercial sizes of pipes and try between the limits established.

For another trial use $d = 22'' = 1.83$; $r = .4575$ $\sqrt{r} = .68$; $C = 103.44$ for .6 and 110.88 for .7; difference 7.44; hence add $8 \times .74 = 5.92$ to $103.44 = 109.36$; trying again formula 3 using factor $C = 109$ we get: $620000 \div 327916 = 1.89$ ft.

This shows that 22 inches is not quite sufficient but is very close.

Using 23 inches $= d$; this is 1.917 ft.; $r = .4792$; $\sqrt{r} = .69$; $C = 110$.

Then $d = 620000 \div 333960 = 1.86$ ft.

Reducing this to inches multiply by 12 gives 22⅓ inches.

5. *Find Length of Cast-Iron Pipe.*

A 48-inch cast-iron water main leads from a reservoir, the water level of which is 100 ft. above the supply end of the pipe; what is the length of the pipe if the velocity is 6.5 ft. per second and $n = 0.12$?

Solution—$r = 1.0$ and $\sqrt{r} = 1.0$, so $C = 127.42$. Change the 100 ft. head into pressure by dividing 2.309 into 100 = 43.31 lbs. Use formula four in Article xv.

$l = 2.3\,p\,d\,C^2 \div 4\,v^2$.

$l = 100 \times 4 \times 127.42 \times 127.42 \div 4 \times 6.5 \times 6.5$.

$l = 38{,}426$ ft.

6. *Find the Factor C for Cast-Iron Pipe.*

Problem—Find the Factor *C* for a 48-inch cast-iron pipe 38,426 ft. long having a head of 100 ft. and a mean velocity of flow of 6.5 ft. per second, if $n = .012$.

This problem is to check Formula 5 by using data of the preceding problem.

$$C = \frac{\sqrt{4 \times 38426 \times 6.5 \times 6.5}}{100 \times 4} = 127.42.$$

This checks correctly with Table IV.

It should be observed in the preceding problems that in determining the factor *C* it was taken from under the column $s = .001$. This is usually correct for pipes flowing full under pressure, as the equivalent head divided into the length give high slopes; mostly higher then .001; should, however, the pressure become so low as to produce slopes corresponding to .0004, .0002, .0001, etc., then the factor *C* must be selected accordingly.

ARTICLE XVI

LOSS AT HEAD BY ENLARGEMENT OF CHANNEL

1. *The Principle Involved.*

Whenever the form of a channel is suddenly enlarged, as indicated in figure 92, there occurs a loss of head along AB which may be represented by the formula: $L = \left(\dfrac{v_1 - v_2}{2\,g}\right)^2$

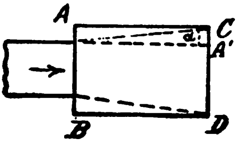

Fig. 92.

in which L is the loss of head in feet, $v_1 =$ the velocity in feet per second in the smaller section and v_2 the velocity in the enlarged section, and $g = 32.16$. If the change from one section to the other is made gradually as indicated by the dotted lines AC and BD the loss of head is very much reduced; if alpha is the angle which AC forms with the produced line then $L = \left(\dfrac{v_1 - v_2}{2\,g}\right)^2 \textit{sin.}\ \alpha$

2. *Applied Problem*—A 12-inch cast-iron water pipe is enlarged to an 18 pipe; if the enlargement is made in 4 ft. and if the velocity in the 12-inch pipe is 6 ft. what is the loss of head?

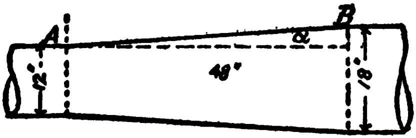

Fig. 93.

Solution—Make sketch as shown in Fig. 93 and apply formula by substituting the given quantities: $AB = \sqrt{48^2 + 3^2} = \sqrt{2315} = 48.11$; then $\textit{sin}\ \alpha = \dfrac{3}{48.11} = .0625.$

The velocity in the 12" section $= 6$ ft. $= v_1$.

The velocity in the 18" section $\dfrac{6}{2.25} = 2.67 = v_2$.

Then $L = \dfrac{(6-2.67)^2 \times .0625}{64.32} = .0118$ ft.

If the change had been made suddenly, *i. e.*, if angle $\alpha = 90°$ then L would be equal to 1.73 ft.

ARTICLE XVII

SUBDIVISIONS OF CHANNELS

1. *The Analysis.*

To divide cylindrical channels into equivalent smaller channels of equal diameters:

Let d = diameter of large conduit.
Let x = diameter of small unit.
Let n = number of small branches.

$$\text{Then } x = \sqrt[5]{\left(\frac{\sqrt[2]{d^5}}{n}\right)^2} \text{ or}$$

$$\text{Log } x = \frac{2}{5}\left(\frac{5 \log d}{2} - \log n\right).$$

This formula may easily be worked back so as to find the size of a large cylindrical pipe equal in hydraulic capacity to a number of equal smaller ones.

Let d be the required diameter, let x be the diameter of the small pipes and n their number, then:

$d = \sqrt[5]{(n\sqrt{x^5})^2}$ or

$$\log d = \frac{2}{5}\left(\frac{5}{2} \log x + \log n\right)$$

2. *Problem*—Apply preceding formula to divide a 16-ft. diameter conduit into 3 equivalent cylindrical channels as indicated in Fig. 94:

$$\log x = \frac{2}{5}\left(\frac{5 \log 16}{2} - \log 3\right)$$

log 16 = 1.204120
5

2: 6.020600 = 3.010300
.477121 = log 3

2.533179
2 multiply

divide by 5) 5.066358 (1.013271

which is the log of x, this gives 10.24 ft. as the diameter of each of the 3 smaller channels.

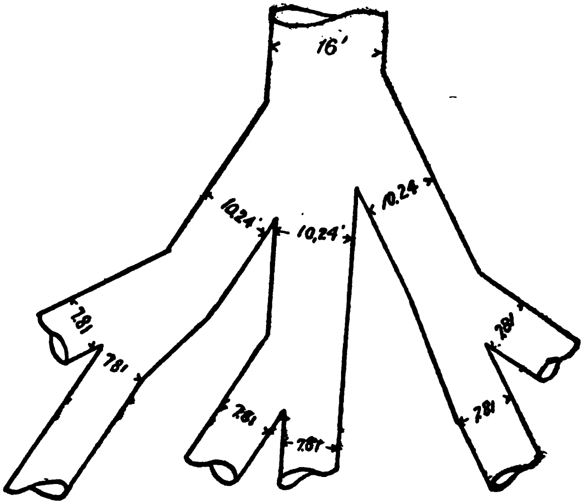

Fig. 94.

Should it be required to split each of the 3—10.24 ft. channels once more, each, say, into 2 channels and let y be the required diameter; then

$$\log y = \frac{2}{5}\left(\frac{5 \log 10.24}{2} - \log 2\right)$$

$$\begin{array}{r} \log 10.24 = 1.013271 \\ 5 \\ \hline 2\,)\,5.066355 = 2.533177 \\ 301030 = \log 2. \\ \hline 2.232147 \\ 2 \\ \hline 5 : 4.464294\ (\ .892859 \end{array}$$

Hence $\log y = .892859 = 7.814$.

To check back find diameter of cylindrical channel which has the same hydraulic capacity as 6 channels each 7.814 ft. in diameter—use formula:

$$\log d = \frac{2}{5}\left(\frac{5}{2}\log x + \log n\right)$$

$$\begin{array}{r}
\log 7.814 = .892859\\
5\\
\hline
2\ )\ 4.464295\ (\ 2.232147\\
0.778151 = \log 6\\
\hline
3.010298\\
2\\
\hline
5\ )\ 6.020596\ (\ 1.204119
\end{array}$$

This is the log of 16 and therefore checks the work as correct.

ARTICLE XVIII

LOSS OF HEAD AT ENTRANCE TO PIPES

If a pipe projects into a tank or reservoir there is a loss of head due to the entrance equal to $L = \frac{v^2}{2g}$; if the pipe does not project the loss of head is just $L = \frac{v^2}{4g}$ or just half of the former; in this formula L = loss of head in ft., v = mean velocity in pipe and $g = 32.16$. This loss is small and is usually neglected except for great velocities. Thus if $v = 4$ ft. the loss = .25 for pipes extending into the reservoir and .125 ft. for pipes just opening into the reservoir. Should the velocity, however, reach 8 ft. per second then the loss of head in the first case would be 1 ft. and in the second ½ ft., quite appreciable quantities which grow as the square of the velocity. Hence, for high velocities the entrance of pipes into tanks should be made flaring, as indicated in figure 95. A good proportion for the enlargement when d = diameter of pipe, make $e = 5d$ and $f = 2d$.

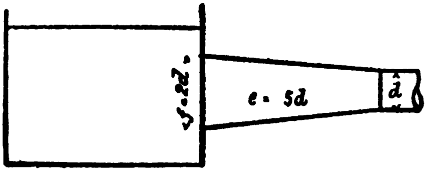

Fig. 95.

ARTICLE XIX

DITCHES

1. *General Remarks.*

The commonest artificial hydraulic channel is the ditch, and the knowledge regarding proper construction is very limited. Yet the adaptability of this kind of channel is very great and is becoming more important on account of the increased use of ditches in irrigation work. For clean, well cut ditches a factor of roughness of .025 is recommended, though for particularly smooth work .020 may be found correct. For rougher work where there are noticeable changes in cross section and grade n should be taken .030 and for ditches that are very irregular and rough in line grade and finish with obstructions to the flow the factor n should be taken .035 to .040 and according to aggravated condition the factor may even have to be taken still greater.

2. *Triangular Ditches.*

The forms of ditches are generally either triangular, rectangular or trapezoidal, the hydraulic principles of which forms have been analyzed in IX-4. In Figs. 96, 97 and 98 are shown typical sections of triangular ditches; Fig. 96 shows side slopes 1 to 1, Fig. 97 shows slopes 1½ to 1 and Fig. 98 shows a slope of 1 to 1 on the left and a slope of 1½ to 1 on the right side. They are all easily analyzed as regards their hydraulic properties. The triangle ABC in Fig. 96 is right angled and if the line CD is drawn perpendicular to AB then $CD = AD$ and the area of $ABC = AD \times DC$. Call $DC = d$ the area $ABC = d^2$, which represents the flow area in such a triangular ditch with a depth of flow of d. The wetted perimeter $= AC + BC$ $AC = \sqrt{2d^2} = d\sqrt{2}$, hence let $P =$ wetted perimeter, then $P = 2d\sqrt{2} = 2.828d$.

From which it follows that $r = d^2 : 2.828d = d \div 2.828 = .353\,d$. From this the $\sqrt{r}$ is easily computed or looked up in the table and the factor C is then found in the tables, reference being had to the slope and the coefficient of roughness as indicated in the preceding paragraph.

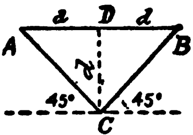

Fig. 96.

In designing ditches it must be remembered that the velocity of the expected flow has to be well considered as the velocity may become too great and destroy the ditch by erosion. Here the hydraulic engineer must use good judgment and if the material of the ditch is easily pitted by the current, then the velocity of the current must be reduced by decreasing the slope and increase the cross section so as to get the required capacity.

If this, however, cannot be done the ditch must be lined either with concrete, boards or paving, in which case the size of the ditch may be reduced, as the smooth lining decreases the resistance to flow and the factor n may be used of a lower value according to the smoothness of the lining.

For the case shown in Figure 97 the area: $a = d \times 1.5\,d = 1.5\,d^2$ and the wetted perimeter $p = 2\sqrt{d^2 + 1.5\,d^2} = 2\sqrt{d^2\,(1 + 2.25)} = 2d\sqrt{3.25} = 3.606\,d$, hence $r = \dfrac{1.5\,d^2}{3.606\,d} = .415\,d$.

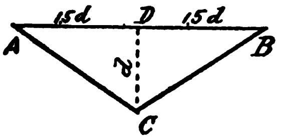

Fig. 97.

For the case shown in Fig. 98, $a = \dfrac{1.5\,d^2}{2} + \dfrac{d^2}{2} = \dfrac{2.5\,d^2}{2} = 1.25\,d^2$; $p = 1.803\,d + 1.414\,d = 3.217\,d$, hence $r = 1.25\,d^2 \div 3.217\,d = .387\,d$—so when the depth of flow is given, a, p and r can be readily computed.

3. *Rectangular Ditches.*

In Fig. 99 is shown a typical section of a rectangular ditch *ABCD* in which *BC*, is the width, and *A B*, the height—let $BC = b$ and $AB = d$ then $bd =$ flow area and $b + 2d =$ wetted perimeter; hence $r = \dfrac{bd}{b + 2d}$ from which the factor C can be found.

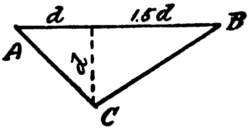

Fig. 98.

Ditches of this cross section are not practicable except in stone cutting or where the ditch is lined with masonry or lumber.

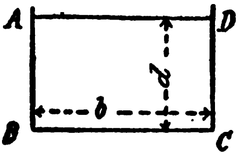

Fig. 99.

4. *Trapezoidal Ditches.*

Fig. 100 shows a ditch with slopes ½ to 1, Fig. 101 shows slopes 1 to 1, Fig. 102 shows slopes 1½ to 1 and Fig. 103 shows slopes 2 to 1. The hydraulic radius is easily computed for each case; let b be the width of base and d the depth of flow in each case, then for Fig. 100 the flow area $a = bd + \frac{d^2}{2} = \frac{d}{2}(2b + d)$; the wetted perimeter $= b + d\sqrt{3}$, hence the hydraulic radius $r = \frac{d}{2}(2b + d) \div b + 1.7321d$.

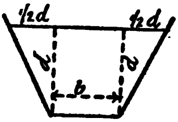

Fig. 100.

In Fig. 101 the flow area $a = bd + d^2 = d\ (b + d)$, the wetted perimeter $= b + 2d\sqrt{2} = b + 2.828d$, hence $r = d\ (b + d) \div b + 2.828d$.

In Fig. 102 flow area $a = bd + 1.5d^2 = d\ (b + 1.5d)$. The wetted perimeter $= b + 2\sqrt{2.25d^2 + d^2} = b + 3.606d$. In Fig. 103 the flow area $a = bd + 2d^2 = d\ (b + 2d)$. The wetted perimeter $= b + 4.4722d$, the hydraulic radius $r = d\ (b + 2d) \div b + 4.4722d$.

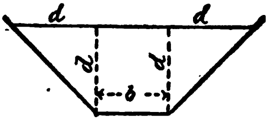

Fig. 101.

In case the slopes should be not symmetrical as indicated in Fig. 104, having a 1 to 1 slope on the left and a 1½ to 1 on the right. This should be treated as shown in Art. XIX. (See Fig. 98.) The flow area $a = bd + \frac{d^2}{2} + \frac{3}{4}d^2 = \frac{4bd}{4} + \frac{2d^2}{4} + \frac{3d^2}{4} = \frac{4bd + 5d^2}{4} = \frac{d}{4}\ (4b + 5d)$.

The wetted perimeter $= b + d\sqrt{2} + d\sqrt{3.25} = b + d\ (\sqrt{2} + \sqrt{3.25}) = b + d\ (1.4142 + 1.8030) = b + 3.2452d$.

Then the hydraulic radius $r = \frac{d}{4}\ (4b + 5d) \div b + 3.2452d$.

ARTICLE XX

DITCH TABLES AND THEIR APPLICATION

1. *How to Use Them.*

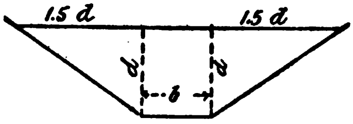

Fig. 102.

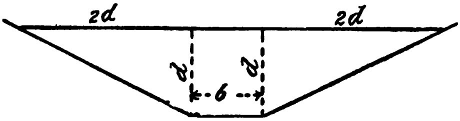

Fig. 103.

The following tables relate to a number of ditches with their hydraulic properties worked out; the factor $n = .025$ and the grades are shown at the head of the various tables. Since the mean velocity varies as the $\sqrt{s}$, everything else being equal, it is easy to form other tables from these given, for any slope desired; for instance, a grade in one of the given tables is 1 in 200, and it is desired to prepare a table for a grade of 1 in 300, then the relation exists:

$$V : v = \sqrt{1:200} : \sqrt{1:300}$$
$$V : v = \sqrt{.005} : \sqrt{.0033.}$$
$$V : v = .07071 : .05746; \text{ hence}$$

$$V = \frac{.05746v}{.07071}$$

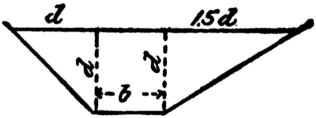

Fig. 104.

This gives the rule to find the mean velocity for any required grade; take the square root of this required grade and multiply with the given velocity; then divide by the square root of the given grade.

2. *Table of Capacity of a Triangular Ditch,* **slope 1 to 1, see Fig. 96, and grade 1 in 400, $n = .025$, $s = .0025$, $\sqrt{s} = .05$.**

Depth of flow d	Flow area a	Wetted Perimeter	Hyd. Rad. r	$\sqrt{r}$	$C.$	$v.$	Cfs.	Gals. per min.
.5	.25	1.414	.177	.42	32	.67	.17	76
1.0	1.00	2.828	.353	.595	41	1.22	1.22	548
1.5	2.25	4.242	.531	.728	46	1.67	3.76	1,687
2.0	4.00	5.656	.706	.842	50	2.10	8.40	3,770
2.5	5.25	7.070	.882	.938	53	2.49	13.07	5,865
3.0	9.00	8.484	1.061	1.03	56	2.88	25.92	11,633
4.0	16.00	11.312	1.414	1.19	60	3.57	57.12	25,636
5.0	25.00	14.140	1.771	1.33	62	4.12	103.00	46,226
6.0	36.00	16.968	2.120	1.45	66	4.78	172.08	77,198

Depth of flow given in feet.

3. Rectangular ditch, with 2-ft. base, grade 1 in 200, $n = .025$, $s = .005$. (See Fig. 99.)

Depth flow.	Flow area.	$W.P.$	$r.$	$\sqrt{r.}$	$C.$	$v.$	$Cfs.$	Gals. per min.
6"	1.0	3.0	.33	.57	38	1.52	1.52	682
12"	2.0	4.0	.50	.72	44	2.22	4.44	1,993
18"	3.0	5.0	.60	.77	46	2.48	7.44	3,338
24"	4.0	6.0	.67	.82	49	2.81	11.24	5,036
36"	6.0	8.0	.75	.87	50	3.05	18.30	8,193
48"	8.0	10.0	.80	.90	52	3.28	26.24	11,776
72"	12.0	14.0	.86	.93	53	3.45	41.40	18,580
96"	16.0	18.0	.89	.94	54	3.55	56.80	25,490
120"	20.0	22.0	.91	.95	54	3.59	71.80	32,222

4. Rectangular ditch, with 4-ft. base, grade 1 in 200, $n = .025$, $s = .005$. (Fig. 99.)

Depth flow.	Flow area.	$W.P.$	$r.$	$\sqrt{r.}$	$C.$	$v.$	$Cfs.$	Gals. per min.
6"	2.0	5.0	.40	.63	42	1.85	3.70	1,660
12"	4.0	6.0	.67	.82	49	2.70	10.80	4,845
18"	6.0	7.0	.86	.93	53	3.45	20.70	9,290
24"	8.0	8.0	1.00	1.00	55	3.85	30.80	13,821
36"	12.0	10.0	1.20	1.10	57	4.39	52.68	23,643
48"	16.0	12.0	1.33	1.15	58	4.67	74.72	33,534
72"	24.0	16.0	1.50	1.22	61	5.20	124.80	44,662
96"	32.0	20.0	1.60	1.26	62	5.47	175.04	78,558
120"	40.0	24.0	1.67	1.29	63	5.69	227.60	102,148

5. Trapezoidal ditch, 1-ft. base, side slopes ½ to 1, grade 1 in 200, $n = .025$, $s = .005$. (Fig. 100.)

Depth.	Area.	r.	$\sqrt{r}$.	$W.P.$	C.	v.	Cfs.	Gals. min.
6″	.63	.29	.54	2.12	38	1.44	.91	408
12″	1.5	.48	.69	3.12	45	2.17	3.25	1,458
18″	2.625	.60	.77	4.36	47	2.53	6.65	2,984
24″	4.00	.91	.95	5.48	53	3.53	14.12	6,337
36″	7.50	.97	.98	7.72	55	3.77	28.28	12,692
48″	12.00	1.20	1.10	9.96	57	4.39	52.67	23,638
72″	24.00	1.66	1.29	14.44	62	5.60	134.4	60,319
96″	40.00	2.11	1.45	18.92	65	6.60	264.0	113,990
120″	60.00	2.56	1.60	23.40	69	7.73	463.8	208,153

6. Trapezoidal ditch, base 1 ft., slope 1 to 1, grade 1 in 800, $n = .025$, $s = .00125$. (Fig. 101.)

Depth.	Area.	r.	$\sqrt{r}$.	$W.P.$	C.	v.	Cfs.	Gals. min.
6″	.75	.31	.56	2.41	38	.75	.56	251
12″	2.00	.53	.73	3.82	46	1.18	2.35	1,055
18″	3.75	.72	.85	5.23	50	1.49	5.58	2,491
24″	6.00	.91	.95	6.64	53	1.79	10.74	4,820
36″	12.00	1.26	1.12	9.46	57	2.24	26.84	12,046
48″	20.00	1.63	1.28	12.28	62	2.78	55.50	24,908
72″	42.00	2.34	1.53	17.92	67	3.59	150.78	69,685
96″	72.00	3.06	1.75	23.56	71	4.35	313.20	140,563
120″	110.00	3.77	1.94	29.20	74	5.02	552.20	247,827

7. Trapezoidal ditch, base 1 ft., slope 1½ to 1, grade 1 in 200, $n = .025$, $s = .005$. (Fig.102.)

Depth.	Area.	r.	$\sqrt{r}$.	$W.P.$	C.	v.	Cfs.	Gals. min.
6″	.88	.31	.56	2.80	38	1.43	1.25	561
12″	2.50	.54	.73	4.61	46	2.35	5.88	2,639
18″	4.87	.76	.87	6.41	50	3.05	14.85	6,661
24″	8.0	.98	.99	8.21	55	3.81	30.48	13,679
36″	16.5	1.39	1.18	11.83	60	4.96	81.90	36,756
48″	28.0	1.81	1.35	15.44	63	5.95	166.60	74,771
72″	60.0	2.64	1.62	22.66	70	7.94	476.40	213,809
96″	104.0	3.48	1.87	29.88	73	9.56	994.24	446,214
120″	160.0	4.31	2.08	37.10	75	10.92	1747.2	784,144

8. Trapezoidal ditch, width of base 4 ft., slope 1½ to 1, grade 1 in 400, $n = .025$, $s = .0025$. (Fig. 102.)

Depth.	Area.	*W.P.*	*r.*	$\sqrt{r}$.	*C.*	*v.*	Cfs.	Gals. min.
2′	14.0	11.2	1.25	1.1	57	3.14	43.96	19,727
4′	40.0	18.4	2.17	1.5	67	5.02	200.80	90,059
6′	78.0	25.6	3.05	1.75	70	6.12	477.36	214,218
8′	128.0	32.8	3.89	1.97	75	7.48	957.44	429,678

9. Trapezoidal ditch, base 1 ft., slope 2 to 1, grade 1 in 800, $n = .025$, $s = .00125$. (Fig. 103.)

Depth.	Area.	*r.*	$\sqrt{r}$.	*W.P.*	*C.*	*v.*	Cfs.	Gals. min.
6′	1.0	.34	.58	3.24	40	.81	.81	363
12′	3.0	.55	.74	5.48	46	1.19	3.57	1,602
18′	6.0	.77	.88	7.72	52	1.60	9.60	4,308
24′	10.0	1.00	1.00	9.96	56	1.96	19.60	8,796
36′	21.0	1.57	1.25	13.44	61	2.67	56.07	25,156
48′	36.0	1.90	1.38	18.92	64	3.08	111.06	49,844
72′	78.0	2.79	1.67	27.88	79	4.09	319.02	143,161
96′	136.0	3.69	1.92	36.84	74	4.97	675.92	303,362
120′	210.0	4.58	2.14	45.80	76	5.69	1194.9	536,571

10. Trapezoidal ditch, 2 ft. base, slope 2 to 1, grade 1 in 1,600, $n = .025$, $s = .00062$. (Fig. 103.)

Depth.	Area.	*W.P.*	*r.*	$\sqrt{r}$.	*C.*	*v.*	Cfs.	Gals. per min.
2′	12.0	11.0	1.01	1.0	56	1.4	16.80	7,539
4′	40.0	20.0	2.00	1.4	65	2.27	91.00	40,840
6′	84.0	29.0	2.87	1.7	70	3.01	253.26	113,612
8′	144.0	38.0	3.75	1.9	73	3.51	506.16	227,150
10′	220.0	47.0	4.64	2.1	76	3.99	877.80	393,957
12′	312.0	56.0	5.58	2.3	78	4.48	1399.32	628,014
14′	420.0	65.0	6.46	2.5	81	5.06	2125.20	975,289

11. Trapezoidal ditch, base 3 ft., slope 1 in 1, grade 1 in 400, $n = .025$, $s = .0025$. (Fig. 101.)

Depth.	Area.	*W.P.*	*r.*	$\sqrt{r}$.	*C.*	*v.*	Cfs.	Gals. min.
2′	10.0	8.6	1.16	1.1	57	3.13	31.30	14,044
4′	28.0	14.2	1.97	1.4	65	4.55	127.40	57,177
6′	54.0	19.8	2.77	1.7	71	6.03	325.62	146,138
8′	88.0	25.4	3.47	1.9	74	7.03	618.64	277,645

12. Trapezoidal ditch, base 10 ft., slope 1 to 1, grade 1 in 1,600, $n = .025$, $s = .00062$. (Fig. 101.)

Depth.	Area.	*W.P.*	*r.*	$\sqrt{r}$.	*C.*	*v.*	Cfs.	Gals. min.
2′	24.0	15.6	1.54	1.2	61	1.83	43.92	19,711
4′	56.0	21.2	2.65	1.6	69	2.76	154.56	69,366
6′	96.0	26.8	3.57	1.8	72	3.24	311.04	139,577
8′	144.0	32.4	4.45	2.1	75	3.75	540.00	242,352
10′	200.0	38.0	5.26	2.3	78	4.48	897.00	402,573
12′	264.0	40.8	6.47	2.5	81	5.06	1335.84	559,525

13. *Advantage of Lined Ditches.*

When conducting water through ditches many things must be considered, such as the maximum and minimum flow to be expected and the resulting velocities. Small flows of moderate velocities can be accommodated by any ditch cut into the natural soil so long as the mean velocity remains below 2 ft. per second. But as the velocity grows, which is always a natural consequence of increased volume, there appears the danger of erosion or the water cutting into the wetted surface of the ditch. Hence in designing a ditch the maximum quantity of water to be dealt with must be carefully determined and also the maximum velocity; if it is found that such maximum velocity would cause the cutting away of the bed in the ditch then it will be better to line the ditch with masonry, brick, concrete, lumber, metal, etc. In adopting this course the coefficient of roughness n is greatly reduced according to the material used, which increases the factor C in proportion so that the dimensions of the ditch may be decreased, or else the slope made smaller, which results in a saving in the required head. Thus an irrigation ditch, for instance, lined with concrete will be able to stand a velocity of 6 ft. per second without injury, while a plain ditch cut through the soil would fail with a current of 2 ft. per second or more. Another advantage of the lined ditch is that there are no losses due to seepage and the loss due to evaporation is much smaller than in the rough ditch.

As the scouring takes place in the bottom of the ditch or water course, the bottom velocity, or the velocity along the wetted perimeter, must be considered, not the mean velocity which is greater than the bottom velocity. By many experiments made for this purpose Kutter found that the bottom velocity is equal to the mean velocity divided by 1.31; thus if $v = 2.0$ ft. per second the velocity on the bottom and sides of the ditch is $2 \div 1.31 = 1.527$ ft. It is the bottom velocities which must be considered in finding the limiting value of velocities permissible in a water course cut in soil.

The following table shows velocities just necessary to move the various substances lying loose on the bed:

	Mean Velocity	Bottom Velocity
Mud and fine clay	0.33 fs.	0.25 fs.
Fine sand	0.46 fs.	0.35 fs.
Clay loam and sand	0.66 fs.	0.50 fs.
Common river sand	0.92 fs.	0.70 fs.
Gravel	2.62 fs.	2.00 fs.
Pebbles, 1 in. in diameter	2.79 fs.	2.13 fs.
Coarse gravel	3.93 fs.	3.00 fs.
Angular broken stone	5.24 fs.	4.00 fs.
Soft slate, shingle stone	6.55 fs.	5.00 fs.
Stratified rock	7.86 fs.	6.00 fs.
Hard rock	13.12 fs.	10.00 fs.

It is seen from this table that it requires but a moderate velocity to move such loose particles as clay or loam, whereas a ditch lined with concrete, for instance, can withstand a bottom velocity of 6 ft. per second or a mean velocity of 7.86 ft. per second.

The above velocities are just sufficient to move particles as indicated. The resistance of soils to erosion is greater as the cohesion in the material has to be overcome; the limiting velocities as to erosion are as follows:

	Feet per Second
Channel in pure sand, bottom velocity	1.1
Channel in sandy soil, 15% clay	1.2
Channel in sandy loam, 40% clay	1.8
Channel in loamy soil, 65% clay	3.0
Channel in clay loam, 85% clay	4.8
Channel in soil clay, 95% clay	6.2
Channel in pure clay, 100% clay	7.3

It is, however, not good policy to permit velocities as high as these unless the water is perfectly clear, since particles in suspension, such as grains of sand and gravel will cut into the walls of the channels.

14. *Flumes.*

Flumes may be considered lined ditches and as such what has been said about ditches applies also to flumes. The essentials of a flume is its form and the material of which it is built, which determines the factor of roughness, the hydraulic radius and the slope. In the building of open flumes it is important to plan the whole work so that the inclination of the slope of the flume is on the same angle as the average hydraulic grade line, taking a mean between high and low water stages. In many irrigation projects it is important to use the smallest slope possible in order to increase the acreage available for irrigation. In order to do this and at the same time to obtain the largest amount of water practicable it is essential to build a flume in which the coefficient of roughness is as small as possible consistent with good economic practice.

For instance, let it be required to design a flume capable of carrying 40 second feet of water a distance of three miles on a grade of 18 in. per mile.

Analysis—Let the size of the flume be 3 ft. by 4 ft. and the flow area be $3 \times 4 = 12$ sq. ft. when delivering 40 second feet; then $v = 40 \div 12 = 3.333$ fps.; then, according to the fundamental formula: $v = C\sqrt{rs}$, we know all the quantities except C, hence by transposition:

$C^2 = v^2/rs$ or $C = \sqrt{v^2/rs}$.

The hydraulic radius is $12 \div 10 = 1.2$ and $\sqrt{r} = 1.09$.

Substitute given quantities and make computations:

$C = \sqrt{3.333^2/1.2 \times .00028} = \sqrt{11.11/.000336} = \sqrt{.33065} =$ 181.8. Hence the factor C is 182 for a hydraulic radius of 1.2 and sine of slope of .0003, which indicates by consulting Tables A and I that the coefficient n must lie between .008 and .009. which means that the flume requires to be almost perfect, and as a consequence it would be cheaper to either enlarge the flume or increase the sine of the slope. As a usual thing this latter alternative is not practicable, so the size of the flume must be increased to reduce the velocity. By making the size of the flume 4x4 gives a flow area of 16 ft.; divide this into 40 gives $v = 2.5$; the hydraulic radius is $16 \div 12 = 1.33$; $\sqrt{r} = 1.16$ and $s = .00028$, and by substituting values in the formula: $C = \sqrt{v^2/rs}$ we obtain:

$C = \sqrt{6.25/1.333 \times .00028} = \sqrt{16756} = 129.4$.

This value of C is considerably lower, and looking in Table IV it is seen that the coefficient $n = .012$ would be fitting the construction of the flume, as a slope of .0004 and a hydraulic radius of 1.0 gives $C = 126$, and for a hydraulic radius of 1.44 $C = 134.29$. Then consult Article X for the selection of construction material.

Should it be possible to increase the slope to say .0004 the size of the flume might be decreased and the size 3 ft. by 4 ft. will give good results as $C = \sqrt{3.333^2/1.2 \times .0004} = \sqrt{11.11/.00048} = \sqrt{23145.8} = 152.1$. By consulting Tables II and III it is seen that the factor *n* lies between .010 and .011, somewhat nearer to .010, which is a guide for the selection of flume material as indicated in Article X.

It is necessary to use good judgment in the planning of hydraulic problems and it is much better to make trial plans and computations for all possible conditions and then select the best, than to find after the work is done that it is not satisfactory.

ARTICLE XXI.

FLOW MEASUREMENTS

How to Measure the Flow Area in an Open Channel.

Let Fig. 105 represent the cross-section of flow in an irregular channel or river bed. It is important to obtain the cross-sectional area of the flow and also the length of the wetted perimeter as exact as possible in order to obtain the true hydraulic radius. It is further necessary to obtain a value for the mean velocity of the flow in the channel. Thus in Fig. 105 the area enclosed by the horizontal AB (water level) and the irregular channel line $AJKLMNOB$ must be determined. This is best done by stringing a line across the stream like AB when the depth is not too great for the observer to wade across the channel—for large rivers cables and bridges will be necessary. Divide the string AB, say into eight equal parts, by points C, D, E, F, G, H, I and with sounding rods find the depths CJ, DK, EL, FM, GN, HO and IP. These lines cut the cross-section into two triangles ACJ and BIP and six trapezoids in between the two triangles, and as the line AB is known with the depth of the seven points above the bottom of the river the area can be readily computed; for example, let $AB = 48$ ft., then $AC = CD = DE = EF = FG = GH = HI = IB = 6$ ft.; let $CJ = 9$ in., $DK = 15$ in., $EL = 21$ in., $FM = 27$ in., $GN = 24$ in., $HO = 18$ in., $IP = 12$in.

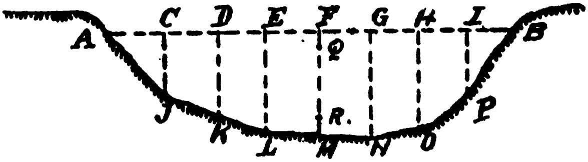

Fig. 105.

Then area of—			Sq. Ft.
Triangle	ACJ	$= 6 \times .75 \div 2 =$	2.25
Trapezoid	$CDKJ$	$= 6\ (.75 + 1.25) \div 2$	6.00
Trapezoid	$DKEL$	$= 6\ (1.25 + 1.75) \div 2$	9.00
Trapezoid	$ELFM$	$= 6\ (1.75 + 2.25) \div 2$	12.00
Tropezoid	$FMGN$	$= 6\ (2.25 + 2.00) \div 2$	12.75
Trapezoid	$GNHO$	$= 6\ (2.00 + 1.5\ ) \div 2$	10.50
Trapezoid	$HOIP$	$= 6\ (1.5\ + 1.0\ ) \div 2$	7.50
Triangle	BIP	$= 6 \times 1 \div 2$	3.00
Total area			63.00

The area of the flow section is consequently 63 sq. ft. To find the wetted perimeter determine the length of the sides *AJ, JK, KL, LM, MN, NO, OP* and *PB*. Considering them as straight lines and using the 47th problem of Euclid for their solution we get:

$AJ = \sqrt{6^2 + .75^2} = \sqrt{36.5625} =$ .... 6.05 ft.
$JK = \sqrt{6^2 + .5^2} = \sqrt{36.25} =$ .... 6.02 ft.
$KL = \sqrt{6^2 + .5^2} = \sqrt{36.25} =$ .... 6.02 ft.
$LM = \sqrt{6^2 + .5^2} = \sqrt{36.25} =$ .... 6.02 ft.
$MN = \sqrt{6^2 + .25^2} = \sqrt{36.0625} =$ .... 6.01 ft.
$NO = \sqrt{6^2 + .5^2} = \sqrt{36.25} =$ .... 6.02 ft.
$OP = \sqrt{6^2 + .5^2} = \sqrt{36.25} =$ .... 6.02 ft.
$PB = \sqrt{6^2 + 1^2} = \sqrt{37.00} =$ .... 6.08 ft.

Total wetted perimeter.............. 48.24 ft.

Therefore the hydraulic radius equals $63.00 \div 48.24 = 1.306$.

2. *How to Find Mean Velocity of Flow.*

The mean velocity is best found by floats or current meters. In channels of regular cross-sections, such as canals, flumes or other artificial channels, floats are suitable means to observe the surface velocity. It is important to take notice of the direction of the wind, as the surface velocity is affected very much by the direction and velocity of the wind; it is therefore best to make these observations when there is no wind.

The maximum velocity in a channel is found at a point about .2 of the whole depth below the surface. Thus if in Fig. 105 the depth $FM = 6$ ft. then at a point *Q*, which is 1.2 ft. below the surface, the flow has its greatest velocity; from point *Q*, either up or down, the velocity decreases, becoming a minimum at point *M* on the bottom. When using current meters to determine the velocity of flow it is usual to take a reading at the .2 point *Q* and another reading at the .8, as at *R* in the line *FM*, add the two readings together and divide by 2; for instance, if the velocity at *Q* is found to be 3.0 ft. per second and at *R* it is 2.0 ft. per second then the mean velocity on line *FM* equals $(3 + 2) \div 2 = 2.5$ ft. These observations or readings must be made in a large number of points and should not be attempted anywhere except in regular channels unless by expert hydraulic engineers.

3. *How to Find Sine of Slope.*

It is not difficult to find the sine of slope in regular channels in which the invert has been built to conform to a definite grade; this applies to ditches, tunnels and flumes of known cross-section and grade. To illustrate let Fig. 106 represent a longitudinal section of a flume or ditch, the line

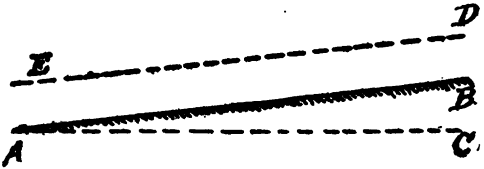

Fig. 106.

AB being the invert line and having a grade of 1 in 1,000; hence if $AB = 1{,}000$ ft., then $BC = 1$ ft., the line AC being horizontal; let a stream of water DE be flowing in this conduit, the depth $DB = 24$ in. and $EA = 25$ in.; the grade of the flow line DE can then be readily found, as it is 1 in. less than the grade of the invert, or 11 in. to the 1,000 ft.; hence by dividing 1,000 into 11 in., or .917 ft., gives $s = .000917$; if the measurement AE had been 22 in. then the slope of the hydraulic grade would be 2 in. greater than that of the conduit or 14 in. per 1,000 ft. or 1.167 which, divided by 1,000, gives $s = .001167$.

When using shorter distances between observations the same rule is observed. If the grade of the line AB is not known it must be established by careful leveling; but as soon as the height of B over A is fixed and the distance AB, the slope of AB is determined by dividing the length AB into BC; for instance, let $AB = 350$ ft. and $BC = 7$ in., then $7/12 \div 350$ gives the slope .00167, and if now the depth of the water at point $B = 24$ in. and at point A 24½ in., then the grade of the water between these two points will be 7½ in. and the sine of the slope will be $7\frac{1}{2} \div (350 \times 12) = .00179$.

4. *Weirs and Weir Measurement.*

One of the principal devices for accurately measuring the flow in a water course, flume or pipe is the *Weir* or *Notch*. They are usually of rectangular cross-section; the lower horizonal edge AB is called the crest (see Fig. 107, which is a front elevation, and Fig. 108, which is a section parallel with the flow). The quantity Q of the flow over such a weir is found by the following formula in which

Q = cubic feet of flow per second.
b = length of weir in feet along the crest.
h = effective head above crest of weir in feet; then:

$Q = 3.33bh^{3/2}$ both end contractions suppressed.

$$Q = 3.33 \left(b - \frac{h}{10}\right) h^{3/2} \text{ one end contraction suppressed.}$$

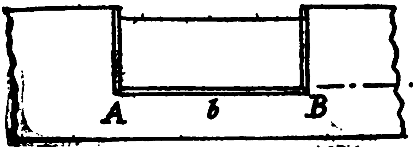

Fig. 107.

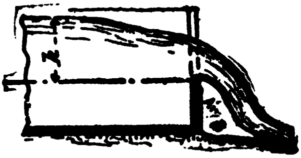

Fig. 108.

$$Q = 3.33 \left(b - \frac{h}{5}\right) h^{3/2} \text{ full end contraction.}$$

These formulae must, however, be used with care as they have been compiled from experiments with weirs of certain definite proportions; the breadth b of the weir should be from three to six times the depth. The distance h should be measured above the fall a sufficient distance to have quiet water, say from 6 to 7 ft. above the weir.

Applied Problem—What is the discharge in cubic feet per second over a weir with full contraction in which the crest $AB = 5$ ft. and the head $h = 15$ in.

Solution—In the formula $Q = 3.33 \left(b - \frac{h}{2}\right) h^{3/2}$ substitute the given values:

$$Q = 3.33 \left(5 - \frac{1.25}{2}\right) 1.25^{3/2} = 3.33 \times 4.375 \times 1.25^{3/2}$$

The fractional exponent 3/2 means the number 1.25 is to be raised to the third power, or multiplied three times by itself and then the square root is to be extracted and that result is to be multiplied by 3.33×4.375. The cube of $1.25 = 1.953125$ and the square root of $1.953125 = 1.394$, hence: $Q = 3.33 \times 4.375 \times 1.394 = 20.309$ cfs.

The cube of 1.25 and the extraction of the square root could have been also done by logarithms, as follows:

Look up in Table of Logarithms 125, which gives the mantissa .09691; since 1.25 is smaller than 10 the characteristic is 0, hence log $1.25 = 0.09691$; multiply by 3 gives 0.29073 as log of 1.25^3; divide by 2 gives 0.145365, which is the logarithm of the required number. Look in Table of Logarithms until you find the mantissa .145365; opposite number 139 under column 7 we find 145196 corresponding to number 1397; this value is slightly too small; the logarithm corresponding to 1398 is 145507, a difference of 311; the difference between 145196 and 145365 is 169; hence as often as 31.1 is contained in 169 that many units in the next lower series are to be added to 1397; hence the next number is a 5, and the answer for $1.25^{3/2} = 1.3975$.

But we could have obtained the whole result by logarithm by adding the logarithms of 3.33 and 4.375 to 0.145196, thus:

Log 3.33 =...................... 0.518514
Log 4.375 =...................... 0.640978
Log $1.25^{3/2}$ =...................... 0.145196

1.304688

Look up number corresponding to logarithm 304688 and we find opposite 201, under column 6, the logarithm 304491, which is the nearest number, 2016, and, as the number is bigger than 10, the decimal point must be placed after the 20, thus 20.16 cu. ft. is the answer.

5. *Measuring Boxes.*

For irrigation purposes many devices are used now under the name of measuring boxes to determine the quantity of water passing a given point. Such boxes are usually made of wood and have been tested for their capacity under different heads before they are placed into commission. The principle involved is illustrated in Fig. 109, which is an end elevation, and Fig. 110, which is a longitudinal section through the box. The rectangular channel *AB* is blocked off by a board *BC*, in which there is a rectangular opening, *DEFG*; this can be closed by a gate so that it will let no water through; if the gate is removed, how-

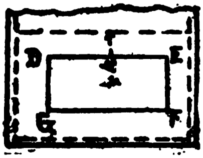

Fig. 109.

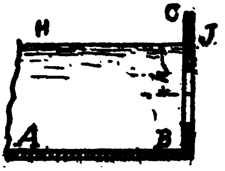

Fig. 110.

ever, the water will flow through the hole *DEFG* at a rate dependent upon the head or the height of the water in the box above the center of gravity of the opening *DEFG;* the center of gravity in a rectangle lies in the intersection of the diagonals or half way down between the horizontal lines *DE* and *GF;* this head can be readily measured with a rule; for instance, let *HJ* be the surface of water in the box and let h be the height of the water above the center of the opening and let a equal the area of the opening. The theoretical velocity of the resulting flow will be the same as if the water had fallen freely a height of h feet and according to the laws of gravity is equal to $\sqrt{2gh}$, or by assuming $2g = 64$, the mean velocity $v = 8\sqrt{h}$; since the area of the flow is a sq. ft. the theoretical quantity Q of discharge is:

$$Q = 8a\sqrt{h}$$

As a matter of fact the actual quantity is considerable less than the theoretical depending upon the form of the opening and box; hence a coefficient C embodying these reducing influences must be incorporated into the formula; this coefficient is found by actual tests of the device under different heads and is then the governing factor for that particular box; this factor C varies from .66 to .60, and in a large number of trials C has been found $=.62$.

Problem—What is the discharge through a measuring box 20x10 in. opening and the height of water 3 in. above top of opening if $C = .62$?

Solution—$h = 3 + 5$ in. $= 8$ in. $= .67$ ft.

The area of opening equals 20 in × 10 in. = 200 sq. in. = 1.39 sq. ft., then

$Q = .62 \times 8 \times 1.39 \times \sqrt{.67} = 6.86 \times .82 = 5.625$ cu. ft. per second.

6. *The Miners' Inch.*

This unit for measuring water developed during the gold excitement in California in 1848 when much of the yellow metal was obtained by washing placer sand and as water was an indispensable article it was measured out to the miners at certain fixed rates. It really amounted to a measuring box having a rectangular hole cut into a 2-in. plank, which blocked off the stream of water, and the water would flow through the orifice under a head of water as indicated in Figs. 109 and 110.

The miner's inch as originally adopted is the amount of water which will flow through a hole 1 in. square in a plank 2 in. thick under a head of 6 in. of water to upper edge of opening. Thus referring back to Fig. 109, let opening $DEFG = 1$ sq. in. and $h = 6\frac{1}{2}$ in.; divide 6.5 by 12 gives .54 ft., hence by using formula:

$Q = 8ac\sqrt{h}$ we find by substitution:

$Q = 8 \times 1/144 \times .62 \sqrt{.54} = 8 \times .007 \times .62 \times .73 = .025$ cfs.

This would be 1.5 cu. ft. per minute or 11.2 gals. per minute.

This is the quantity of California miner's inch according to the law of March 23, 1901, so that 40 miner's inches equal 1 second foot.

The Arizona miner's inch is the same as that of California.

The proportion of the Colorado miner's inch may be best expressed by saying that 38.4 Colorado miner's inches are 40 California miner's inches.

The miner's inch as a measuring unit is, however, doomed to oblivion, as practically all modern measurements are expressed in cubic feet per second, or second feet, and this is certainly a move in the right direction.

ARTICLE XXII

THE USE OF LOGARITHMS

The purpose of these tables are to be used where it is required to multiply, divide, raise to powers or to extract roots, and are based on the principle that multiplication is accomplished by adding, division by subtraction, the raising to powers by multiplication, and the extraction of roots by division of logarithms.

So these four operations will be illustrated now.

1. *Multiplication by logarithms.*

Multiply 69 by 87.

Solution: Look in the table of log and write down the log of 69 = 1.838849; right under that write log of 87 = 1.939519. Add the two numbers, which gives 3.778368 as the log of the required number.

Now look through the table until you find 778368, which you find opposite the number 600 in the column headed 3; so the answer is 6003; the No. 3, characteristic of the log, indicates that the number is greater than 1000 and less than 10000, so the number 6003 is the answer.

Problem: Multiply 13.41 by 81.56.

Solution: Find log of 13.41 in table; look for No. 134 and under column 1 = .127429; the characteristic is 1. Hence, log 13.41 = 1.127429. Find log of 81.56. Look opposite No. 815 under column 6 = .911477; the characteristic = 1, so log 81.56 = 1.911477; add them together; gives log of desired product = 3.038906. The characteristic 3 shows the product is greater than 1000. Look up the number which corresponds to the mantissa .038906; under the nearest smaller .038620 we get the number 1093 and the next higher under 039017 we get the number 1094 so the required number will lie between 1093 and 1094. The difference between the two successive logarithms is 397, or, with other words, while the log grows from .038620 to .039017, the number grows from 1093 to 1094, being a difference of 397 for 1 unit; this would make a rate of 39.7 for each tenth advance. Subtract now .038620 from the log of the required number, 038906, which gives 286; now divide 39.7 into this and the result will be the decimal part of the answer to be added to 1093; this gives .72, hence the answer is 1093.72.

Multiply 171.6 by .043.

Solution: Look up mantissa of log of 171.6 = .234517; look up mantissa of log 43 = .633468. The index of the first log is 2, which place ahead of the decimal; the index of the second logarithm is — 2. Hence, place — 2. ahead of second logarithm and add, which gives 0.867985. Look in tables of log. for the number belonging to .867985; we find for 867935 = 7210; we find for 867995 = 7211; difference 60. Subtract 867935 from 867985, which gives 50; divide 6 in 50, which gives additional numbers = 83, which add to 7210 = 721083. The index is zero, hence the number is less than 10 and greater than 1; hence the answer = 7.21083.

Multiply 23.4753 by .00371.

As the tables give only log for 4 periods, the corrected log is found by interpolation; the log of the first number lies between 2347 and 2348; log mantissa of 2347 = 370513, log mantissa of 2348 = 370689; difference, 185. The mantissa 370513 is to be increased by the product of 53 × 1.85 = 98, making log of 234753 equal to 370611.

The index = 1, hence complete log = 1.370611, log of second number .00371 = —3.569374, add these, which gives —2.939985. Find number belonging to this log. For 939968 we find 8709; for 940018 we find 8710. Difference, .50.

Subtract 939968 from 939985 = 17. Divide 5 into 17 = 3, which add to 8709, making number 87093.

The index is —2, hence the number is smaller than 1/10 and greater than 1/100, hence the decimal point should be placed thus: .087093.

The multiplication by log has been very thoroughly shown to cover all cases that may arise.

2. *Division by logarithms.*

Divide 63 by 9.

Solution: Log of 63 = 1.799341; log of 9 = 0.954243. Subtract, leaves 0.845098. Look up number corresponding to this log. We find 7.

Divide 33.753 by 7.23.

Form log of 33.753 as shown above. Log 33.753 = 1.528313; subtract log 723 = 0.859138 = 0.669175. Find number to this log: 4.6685.

Divide 3517.5 by 28914.0.

Log 3517.5 = 3.546234; log 28914.0 = 4.461108; subtract, —1.085126. Required number = .121654.

In most problems it will not be necessaary to carry as many decimals as this and in most cases the work can be done without interpolation, as the answer .1216 would be all that would be required.

3. *Involution by logarithms.*

Suppose it would be required to raise the number 7 to the 5th power, or multiply 7 —5 times with itself. Look up log of 7 = 0.845098. Multiply by 5 = 4.225490. This is log of required number. Look up this number—16807, answer.

Raise 1.23 to the 7th power.

Log 1.23 = 0.089905.

Multiply by 7 = 0.629335.

Result: 4.259, which should be near enough.

Raise .94 to the 6th power.

Answer: Log .94 = —1.973128.

Multiply by 6 = —1.838768.

It must be observed here that the characteristic is negative —1 and the mantissa is always position; so when multiplying the mantissa by 6 we have 5 positive units to carry to the 6 negative ones which results in the index of —1, so that the resulting number is smaller than 1, but greater than 1/10.

.6899 is the answer.

4. *Evolution by logarithms.*

The operation of extracting roots is called evolution and is accomplished by dividing the log of the radical by the index of the root and finding the number belonging to such logarithms.

Problem: Extract the square root of 64.

Solution: Log 64 = 1.80618. Divide by 2 = .90309, which is log of root, the number which corresponds to this log is 8 —.

Problem: Extract cube root of 125.

Solution: Log 125 = 2.09691.

Divide by 3 = .69897. This is the log of cube root, and the number corresponding to it is 5.

Problem: Extract cube root of 31.29.

Solution: Log of 31.29 = 1.495406.

Divide by 3 = 0.498469.

$\sqrt[3]{31.29} = 3.152$.

Problem: Extract the 5th root of .04782.

This problem is more difficult as the index of the log of .04872 will be —2, which is not divisible by 5, but if we increase —2 by —3 we get —5, which is divisible, and in order to balance the work we must add +3. to the mantissa, thus: log of .04782 = —2.679610, for which we can set: —5 + 3.679610; divide now by 5, which gives —1.735922, and the number required = .5444.

"C" Tables

The following tables have been computed with the idea of giving the value of "*C*", either directly, without figuring, or by a simple interpolation between the given values; they are intended to cover all practical cases of channels, applying to channels having a range in hydraulic radii from 1/100 to 900 ft.; this for cylindrical channels is expressed in the first column of Table VII, where the diameters vary from ½ inch to 3,600 ft.

By the use of these tables the difficult task of finding the factor *C* in Kutter's formula: $v = C\sqrt{rs}$ will be avoided and the solution of hydraulic problems correspondingly simplified.

Table *A* is figured only to an hydraulic radius of 4 ft., as this will fully cover all practical cases in which factor $n = .008$.

Table A.

$n = .008.$

			$s =$ Slopes.			
$\sqrt{R}$	.001	.0004	.0002	.0001	.00005	.000025
.1	59.48	54.24	51.85	45.02	36.74	28.55
.2	97.53	93.39	89.85	78.15	66.00	53.12
.3	123.88	118.87	113.08	103.48	89.77	74.41
.4	143.41	138.58	133.12	123.68	109.65	93.23
.5	158.29	154.67	143.58	139.98	126.38	109.81
.6	170.08	166.83	161.75	153.64	140.90	124.78
.7	179.61	176.93	172.13	164.98	153.28	137.52
.8	187.51	185.03	180.94	174.47	163.88	149.76
.9	194.13	191.29	188.47	182.80	173.42	160.55
1.0	199.79	197.98	194.97	190.07	181.83	170.43
1.2	208.70	207.44	205.56	202.12	196.23	186.64
1.4	216.09	213.99	213.95	211.78	208.00	202.34
1.6	221.57	221.22	220.58	219.53	217.91	214.90
1.8	226.52	226.54	226.57	226.59	226.65	226.72
2.0	229.95	230.22	230.73	231.51	233.03	235.44

Table I.

$n = .009$.

$s =$ Slopes.

$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
.1	49.15	46.47	42.72	36.99	30.51	23.90
.2	81.90	78.37	73.36	65.49	55.37	44.79
.3	105.40	101.61	96.20	87.67	76.08	63.20
.4	122.80	119.31	114.15	105.40	93.44	79.63
.5	136.40	133.24	128.42	120.40	108.33	94.22
.6	147.10	144.49	140.35	132.14	121.05	107.31
.7	156.40	153.73	150.30	143.33	132.30	119.16
.8	163.70	161.48	158.06	152.18	142.38	130.11
.9	170.60	168.12	165.18	160.30	150.25	139.84
1.0	175.40	172.33	171.23	166.20	159.04	148.62
1.2	184.50	183.23	181.26	178.20	172.67	164.82
1.4	191.10	190.33	189.27	186.80	183.43	178.49
1.6	196.50	195.17	196.07	194.90	192.90	190.35
1.8	201.30	200.98	200.97	200.96	200.70	200.67
2.0	204.70	204.90	205.50	206.80	207.60	209.81
2.5	211.80	212.60	214.04	216.70	221.50	229.15
3.0	216.80	217.90	220.10	223.90	232.60	242.90
3.5	220.50	222.10	224.70	229.60	239.20	254.44
4.0	223.30	225.30	228.30	234.10	245.10	263.90
5.0	227.40	229.80	233.20	240.80	254.20	278.15
6.0	230.20	232.80	237.80	245.10	260.70	288.50
7.0	232.40	235.20	240.00	248.50	265.50	296.49
8.0	233.90	236.90	242.30	251.30	269.40	302.81
9.0	235.20	238.10	243.50	253.10	272.50	307.80
10.0	236.10	239.20	244.60	255.60	274.80	311.90
12.0	237.80	241.10	246.50	258.00	278.50	318.40
14.0	238.90	242.30	247.90	258.90	282.00	323.20
20.0	240.80	244.40	250.80	262.70	286.40	332.10
30.0	242.50	246.10	252.50	265.50	290.50	339.50

Table II

$n = 010.$

Slopes.

$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
.1	41.45	39.20	36.06	31.41	25.87	20.43
.2	70.04	66.98	62.52	56.80	47.37	38.52
.3	90.92	87.70	82.89	75.55	65.47	54.67
.4	106.88	103.71	99.02	91.54	80.95	69.09
.5	119.31	116.60	112.14	104.93	94.35	82.13
.6	129.61	126.93	122.86	116.11	106.04	93.85
.7	138.01	135.92	131.88	127.2	116.30	104.72
.8	145.02	142.69	139.60	134.11	125.64	114.77
.9	151.35	149.17	146.32	141.68	133.44	123.65
1.0	156.2	154.59	152.08	148.35	140.95	131.94
1.2	164.6	163.50	161.72	158.72	153.50	146.96
1.4	171.23	170.51	169.45	168.32	164.24	159.60
1.6	176.51	176.19	175.62	174.77	173.25	170.95
1.8	180.84	180.89	180.87	180.81	180.75	180.63
2.0	184.54	184.83	185.25	186.04	187.30	189.32
2.5	191.43	192.40	193.58	196.08	200.64	207.35
3.0	196.94	197.7	199.62	203.59	210.34	221.48
3.5	200.09	201.7	204.2	209.16	218.06	232.71
4.0	202.08	204.86	207.85	213.62	224.20	241.94
5.0	207.08	209.4	213.09	220.19	233.25	256.18
6.0	209.97	212.5	216.67	224.72	239.81	266.57
7.0	212.14	214.9	219.3	228.29	244.65	274.68
8.0	213.65	216.5	221.34	230.72	248.6	281.12
9.0	214.97	217.96	222.97	232.84	251.49	286.40
10.0	216.00	219.08	224.28	234.44	254.00	290.36
12.0	217.46	220.78	226.25	237.04	257.85	296.56
14.0	218.52	222.01	227.7	239.08	260.63	301.90
20.0	220.69	224.49	230.34	242.32	265.90	303.80
30.0	222.30	226.07	231.60	245.14	270.07	318.70

Table III.

$n = .011$.

$\sqrt{r}$	Slopes. .001	.0004	.0002	.0001	.00005	.000025
.1	35.51	33.58	31.37	27.03	23.93	17.76
.2	60.74	58.35	54.24	48.52	41.14	33.64
.3	79.48	76.35	72.45	65.87	57.21	47.93
.4	94.12	91.25	87.07	80.33	71.14	60.49
.5	106.23	103.02	99.08	92.62	83.29	72.58
.6	115.23	112.77	109.05	102.77	93.96	83.41
.7	123.11	120.87	117.47	112.12	103.44	93.17
.8	129.77	127.80	124.77	119.54	111.91	102.22
.9	135.50	133.79	131.12	127.36	119.51	110.63
1.0	140.42	138.92	136.82	132.38	126.39	118.25
1.2	148.56	147.58	145.88	142.97	138.39	132.21
1.4	155.69	154.31	152.99	151.45	148.41	144.18
1.6	160.18	159.91	159.61	158.49	156.89	154.73
1.8	164.46	164.41	164.40	164.33	164.24	164.16
2.0	168.07	168.29	168.64	169.44	170.65	172.49
2.5	174.77	175.67	176.90	179.31	183.53	189.28
3.0	179.76	181.09	182.97	186.62	193.85	203.59
3.5	183.54	184.98	187.43	192.23	200.79	214.71
4.0	186.32	188.10	191.05	196.62	207.57	223.93
5.0	190.41	192.64	196.34	203.19	215.63	237.98
6.0	193.33	195.81	199.89	207.74	222.52	248.55
7.0	195.37	198.16	202.63	211.30	227.50	256.57
8.0	197.03	199.82	204.51	213.79	231.43	262.91
9.0	198.34	201.36	206.24	215.90	234.52	268.19
10.0	199.27	202.42	207.60	216.70	236.92	272.55
12.0	200.87	204.14	209.60	220.25	240.83	279.27
14.0	201.97	205.42	211.00	222.15	243.68	284.24
20.0	204.08	207.71	213.76	225.69	249.03	293.70
30.0	205.68	209.52	215.90	228.43	253.35	301.56

Table IV.

$n = .012$.

Slopes.

Vr	.001	.0004	.0002	.0001	.00005	.000025
.1	30.86	29.19	26.89	23.56	19.53	15.64
.2	53.29	50.93	45.30	42.57	36.24	29.78
.3	70.35	67.75	64.02	58.23	50.63	42.58
.4	83.76	81.14	77.38	71.37	63.22	54.27
.5	94.52	92.08	87.54	82.52	74.29	65.00
.6	103.44	101.15	97.77	92.15	84.12	74.47
.7	110.88	108.81	105.70	100.50	92.89	83.72
.8	117.21	115.41	112.56	107.81	100.80	92.12
.9	122.68	121.07	118.59	114.50	107.95	99.95
1.0	127.42	126.00	123.09	120.10	117.55	106.90
1.2	135.28	134.29	132.72	128.82	125.65	120.04
1.4	141.42	140.82	139.81	138.07	135.35	131.43
1.6	146.55	146.15	145.86	145.00	143.46	141.48
1.8	150.70	150.69	150.65	150.59	150.54	150.40
2.0	154.21	154.44	154.87	155.50	156.77	158.65
2.5	161.01	161.84	163.01	165.26	169.07	175.41
3.0	165.81	167.03	168.96	172.48	178.78	187.94
3.5	170.22	171.01	173.44	178.05	186.15	199.60
4.0	172.38	174.12	176.79	182.46	193.09	208.33
5.0	176.45	178.66	182.19	188.96	201.43	222.64
6.0	179.35	181.85	185.60	193.66	207.98	233.07
7.0	181.49	184.21	188.55	197.37	212.95	241.34
8.0	183.07	185.59	190.58	199.64	217.69	247.74
9.0	184.39	187.33	192.30	201.82	219.99	252.83
10.0	185.43	188.52	193.57	203.58	222.63	257.00
12.0	187.01	190.27	195.59	206.17	226.52	264.25
14.0	188.14	191.54	197.10	208.16	229.33	269.27
20.0	190.58	193.87	199.97	211.71	234.86	279.15
30.0	191.90	195.71	202.05	214.61	239.35	287.17

Table V.

$n = .013.$

Slopes.

$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
.1	27.15	25.68	23.69	20.78	17.29	13.96
.2	47.26	45.17	42.22	37.84	32.23	26.64
.3	62.84	60.49	57.16	52.00	45.27	38.21
.4	75.21	72.83	69.42	64.01	56.73	48.83
.5	85.23	82.56	79.68	74.36	67.06	58.66
.6	93.62	91.51	88.36	83.28	76.63	67.59
.7	100.63	98.77	95.91	91.11	84.19	75.90
.8	106.72	104.98	102.39	98.12	91.54	83.80
.9	111.94	110.17	108.07	104.15	98.42	91.09
1.0	116.48	115.29	113.15	109.65	104.39	97.77
1.2	124.05	123.06	121.62	118.75	115.05	109.67
1.4	130.11	129.46	128.50	126.55	124.05	120.71
1.6	134.96	134.68	134.26	133.31	132.18	130.39
1.8	139.09	139.08	139.07	139.05	138.96	138.84
2.0	142.55	142.80	143.16	143.90	145.00	146.65
2.5	149.25	149.98	151.24	153.43	157.13	162.80
3.0	154.09	155.21	157.09	160.61	166.78	175.71
3.5	157.64	159.17	161.51	166.54	174.00	186.57
4.0	160.58	162.26	165.08	170.25	179.92	195.53
5.0	164.65	166.85	170.28	176.91	189.00	209.50
6.0	167.57	170.06	173.97	181.58	195.83	220.07
7.0	169.69	172.37	176.76	185.18	200.65	227.92
8.0	171.34	174.15	178.81	188.09	204.41	234.64
9.0	172.64	175.59	180.46	189.91	207.87	240.42
10.0	173.63	176.77	181.83	191.63	210.36	244.41
12.0	175.27	178.50	183.36	194.26	214.35	251.16
14.0	176.46	179.75	185.33	196.23	217.27	256.78
20.0	178.56	182.14	188.14	199.88	222.93	266.55
30.0	180.27	184.05	190.39	202.90	227.48	274.89

Table VI.

$n = .015.$

Slopes.

$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
.1	21.56	20.43	18.86	16.62	13.94	11.39
.2	38.14	36.44	34.09	30.58	26.21	21.88
.3	51.29	49.39	44.25	42.45	37.10	31.57
.4	61.95	59.99	57.13	52.69	46.81	40.55
.5	70.83	68.03	66.08	61.61	55.56	48.89
.6	78.27	76.44	73.77	69.44	63.42	56.64
.7	84.59	82.96	80.47	76.39	70.59	63.89
.8	91.18	88.58	86.35	82.54	77.12	70.62
.9	94.93	93.44	91.53	88.09	83.08	77.01
1.0	99.10	97.96	96.11	93.07	88.56	83.00
1.2	106.18	105.32	103.98	101.77	98.33	93.92
1.4	111.89	111.35	110.51	108.96	106.73	103.65
1.6	116.59	116.29	115.91	115.21	113.99	112.42
1.8	120.53	120.52	120.50	120.46	120.39	120.33
2.0	123.91	124.08	124.45	125.13	126.11	127.48
2.5	130.43	131.11	132.23	134.29	138.00	142.80
3.0	135.15	136.25	137.97	141.20	146.75	155.21
3.5	138.58	140.15	142.48	146.61	154.00	165.48
4.0	141.53	143.24	145.91	150.95	159.82	174.12
5.0	145.79	147.79	151.14	157.50	169.00	187.90
6.0	148.66	150.99	153.87	162.18	175.52	198.33
7.0	150.81	153.37	157.61	165.67	180.76	206.57
8.0	152.46	155.20	159.77	168.42	184.70	213.13
9.0	153.76	156.66	161.40	170.63	187.90	218.53
10.0	154.77	157.85	162.75	172.38	190.52	223.18
12.0	156.40	159.65	164.85	175.16	194.80	230.36
14.0	157.61	160.97	166.42	177.13	197.76	235.77
20.0	159.89	163.39	169.33	181.00	203.60	246.24
30.0	161.58	165.34	171.61	183.72	208.32	255.07

Table VII.

$n = .017.$

		Slopes.					
Diameter.	$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
½ in.	.1	19.98	18.77	17.14	14.87	12.29	9.95
2 in.	.2	31.60	30.22	28.30	25.45	21.94	18.49
4⅓ in.	.3	42.90	41.31	39.06	35.59	31.23	26.77
7⅔ in.	.4	52.27	50.59	48.18	44.47	39.62	34.52
12 in.	.5	60.13	58.48	56.11	52.30	47.24	41.78
1.44 ft.	.6	66.83	65.24	62.92	59.21	54.18	48.58
1.96 ft.	.7	72.62	71.14	68.98	65.44	60.54	54.97
2.56 ft.	.8	77.14	76.33	74.30	71.03	66.38	60.97
3.24 ft.	.9	82.02	80.89	79.08	76.09	71.75	66.63
4.00 ft	1.0	85.96	84.93	83.32	80.65	76.72	72.00
5.76 ft.	1.2	92.65	91.87	90.69	88.67	85.65	84.39
7.84 ft.	1.4	98.06	97.55	96.77	95.44	93.42	90.76
10.24 ft.	1.6	102.54	102.29	101.91	101.25	100.17	98.79
12.96 ft.	1.8	106.33	106.32	106.29	106.27	106.24	106.17
16.00 ft.	2.0	109.58	109.78	110.09	110.65	111.58	112.79
25.00 ft.	2.5	115.93	116.59	117.63	119.56	122.68	127.25
36.00 ft.	3.0	120.57	121.63	123.29	126.30	131.45	139.10
49.00 ft.	3.5	124.13	125.51	127.65	131.81	138.49	149.03
64.00 ft.	4.0	126.95	128.55	131.13	135.94	144.31	157.41
100.00 ft.	5.0	132.07	133.14	136.37	142.46	153.33	170.98
144.00 ft.	6.0	134.04	137.22	140.07	147.12	159.95	181.40
196.00 ft.	7.0	136.23	138.76	142.86	150.70	165.07	189.62
256.00 ft.	8.0	137.94	140.64	145.03	153.49	169.14	196.31
324.00 ft.	9.0	139.27	142.09	146.73	155.73	172.43	201.79
400.00 ft.	10.0	140.34	143.29	148.10	157.52	175.18	208.03
576.00 ft.	12.0	142.00	145.13	150.31	160.37	179.44	213.90
784.00 ft.	14.0	143.21	146.48	151.85	162.44	182.62	219.45
1,600.00 ft.	20.0	145.43	149.02	154.81	166.36	188.67	230.37
3,600.00 ft.	30.0	147.22	151.00	157.21	169.49	193.59	239.58

Table VIII.

$n = .020.$

$\sqrt{r}$	Slopes. .001	.0004	.0002	.0001	.00005	.000025
.1	15.19	14.31	13.14	11.49	9.63	7.94
2.	24.81	23.74	22.26	20.10	17.47	14.91
.3	34.07	32.81	31.04	28.36	25.05	21.70
.4	41.91	40.56	38.60	35.72	31.97	28.03
.5	48.60	47.26	45.31	42.28	38.36	34.15
.6	54.42	53.10	51.20	48.23	44.17	39.88
.7	59.48	58.25	56.42	53.58	49.65	45.29
.8	63.97	62.81	61.20	58.45	54.66	50.42
.9	67.95	66.89	65.36	62.82	59.37	55.30
1.0	71.48	70.56	69.26	66.94	63.70	59.93
1.2	77.57	76.90	75.83	74.12	71.62	68.54
1.4	82.52	82.13	81.60	80.28	78.56	76.42
1.6	86.75	86.55	86.40	85.64	84.77	83.60
1.8	90.37	90.36	90.35	90.33	90.26	90.17
2.0	93.48	93.64	93.81	94.42	95.21	96.28
2.5	99.55	100.19	101.16	102.83	105.79	109.59
3.0	104.12	105.11	106.50	109.37	114.00	120.71
3.5	107.64	108.91	110.92	114.62	120.80	130.07
4.0	110.45	111.96	114.31	118.79	126.48	138.15
5.0	114.58	116.51	119.75	125.28	135.40	151.32
6.0	117.52	119.76	123.25	130.05	142.04	161.64
7.0	119.74	122.20	125.91	133.65	147.22	169.85
8.0	121.47	124.08	128.63	136.50	151.31	176.54
9.0	122.83	125.60	130.20	138.68	154.74	182.23
10.0	123.75	126.86	131.63	140.69	157.49	186.96
12.0	125.66	128.74	133.82	143.57	161.93	194.54
14.0	126.91	130.19	135.36	145.81	165.25	200.46
20.	129.28	132.67	138.44	149.77	171.55	211.88
30.	131.07	134.85	140.90	153.09	176.78	221.72

Table IX.

$n = .025.$

			Slopes.			
$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
.1	10.53	9.96	9.21	8.16	6.96	5.88
.2	17.83	17.10	16.09	14.63	12.87	11.18
.3	24.86	23.96	22.73	20.85	18.60	16.37
.4	30.92	29.97	28.50	26.53	23.94	21.31
.5	36.29	35.28	33.86	31.68	28.59	26.03
.6	41.00	40.01	38.60	36.41	33.54	30.50
.7	45.17	44.23	42.87	40.74	37.91	34.84
.8	48.92	48.06	46.77	44.72	41.96	38.97
.9	52.34	51.49	50.30	48.41	45.80	42.88
1.0	55.62	54.64	53.57	51.83	49.39	46.68
1.2	60.69	60.14	59.32	57.96	56.00	53.81
1.4	65.17	64.81	64.24	63.31	61.88	60.37
1.6	69.00	68.82	68.52	67.89	67.28	66.60
1.8	72.27	72.26	72.24	72.22	72.19	72.12
2.0	75.12	75.29	75.35	75.90	76.63	77.41
2.5	80.95	81.47	82.25	83.75	86.06	89.15
3.0	85.28	86.15	87.51	89.96	93.79	99.19
3.5	88.71	89.85	91.65	94.21	100.26	107.83
4.0	91.45	92.85	95.03	99.02	105.68	115.41
5.0	95.64	97.39	100.22	105.41	114.32	128.48
6.0	98.56	100.67	103.99	110.15	120.95	138.08
7.0	100.82	103.15	106.85	113.77	126.20	146.08
8.0	102.62	105.11	109.13	116.71	130.38	152.88
9.0	104.06	106.64	110.90	119.11	133.88	157.26
10.0	105.18	107.98	112.47	121.07	136.72	163.48
12.0	106.96	109.94	114.77	124.11	141.40	171.42
14.0	108.25	111.39	116.51	126.43	144.85	177.55
20.0	110.67	114.12	119.75	130.75	151.72	189.87
30.0	112.70	116.35	122.40	134.33	157.40	200.76

Table X.

$n = .030.$

	Slopes.					
$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
.1	7.81	7.47	6.84	6.22	5.39	4.64
.2	13.67	13.15	12.41	11.35	10.09	8.89
.3	19.26	18.57	17.75	16.32	14.67	13.07
.4	24.19	23.45	22.42	20.88	18.98	17.07
.5	28.57	27.82	26.75	25.11	23.02	20.94
.6	32.53	31.78	30.67	28.98	26.85	24.64
.7	36.10	35.46	34.27	32.59	30.47	28.21
.8	39.29	38.62	37.57	36.05	33.86	31.64
.9	42.25	41.59	40.63	39.28	37.13	34.96
1.0	44.95	44.32	43.46	42.08	40.19	38.14
1.2	49.66	49.19	48.52	47.47	45.92	44.19
1.4	53.66	53.39	52.93	52.17	51.19	49.86
1.6	57.17	57.02	56.81	56.35	55.83	55.14
1.8	60.22	60.20	60.19	60.16	60.14	60.11
2.0	62.87	63.04	63.22	63.58	64.11	64.77
2.5	68.37	68.82	69.53	70.85	72.75	74.97
3.0	72.58	73.34	74.54	76.75	79.94	84.40
3.5	75.87	76.92	78.52	81.44	86.05	92.40
4.0	78.64	79.91	81.88	85.90	91.25	99.44
5.0	82.73	84.37	86.99	91.73	99.65	111.42
6.0	85.75	87.73	90.77	96.46	106.22	121.11
7.0	88.05	90.23	93.68	100.21	111.47	129.13
8.0	89.82	92.21	95.16	103.14	115.68	135.91
9.0	91.26	93.87	97.82	105.60	119.26	141.69
10.0	92.50	95.10	99.38	107.58	122.49	146.62
12.0	94.32	97.24	101.89	110.77	127.17	154.75
14.0	95.66	98.76	103.85	113.20	130.83	161.18
20.0	98.25	101.58	107.10	117.66	138.78	174.15
30.0	100.38	104.00	109.83	122.51	144.07	188.47

Table XI.

$n = .035$.

			Slopes.			
$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
.1	6.18	5.89	5.51	4.89	4.37	3.82
.2	10.95	10.55	10.00	9.20	8.25	7.36
.3	15.55	15.04	14.33	13.30	12.05	10.84
.4	19.67	19.10	18.29	17.10	15.74	14.20
.5	23.40	22.79	21.93	20.65	19.05	17.45
.6	26.78	26.17	25.29	23.96	22.30	20.61
.7	29.87	29.25	28.39	27.07	25.39	23.65
.8	32.68	32.09	31.27	29.98	28.48	26.59
.9	35.26	34.71	33.94	32.73	31.15	29.43
1.0	37.65	37.15	36.49	35.30	33.81	32.19
1.2	41.89	41.50	40.95	40.04	38.82	37.46
1.4	45.47	45.30	44.91	44.29	43.48	42.41
1.6	48.77	48.63	48.43	48.08	47.63	47.09
1.8	51.61	51.58	51.57	51.55	51.53	51.51
2.0	54.10	54.23	54.40	54.71	55.14	55.67
2.5	59.29	59.71	60.36	61.46	63.09	65.21
3.0	63.34	64.04	65.12	66.99	69.87	73.58
3.5	66.62	67.52	68.97	71.58	75.54	81.00
4.0	69.25	70.44	72.62	75.43	80.55	87.64
5.0	73.42	74.93	77.29	81.60	88.72	98.99
6.0	76.35	78.19	81.11	86.33	95.24	108.38
7.0	78.71	80.77	84.02	90.08	100.39	116.23
8.0	80.52	82.82	86.42	93.04	104.73	122.91
9.0	81.99	84.41	88.22	95.54	108.34	128.66
10.0	83.22	85.77	89.82	97.66	111.42	133.70
12.0	85.16	87.82	92.39	100.95	116.34	141.95
14.0	86.55	89.45	94.29	103.44	123.34	148.51
20.0	89.21	92.49	97.87	108.25	127.68	162.04
30.0	91.41	94.97	100.82	112.31	134.24	174.39

Table XII.

$n = .040.$

Slopes.

$\sqrt{r}$	.001	.0004	.0002	.0001	.00005	.000025
.1	5.45	4.82	4.54	4.12	3.65	3.23
.2	9.07	8.75	8.31	7.69	6.95	6.26
.3	12.96	12.55	11.99	11.18	10.19	9.25
.4	16.49	16.02	15.37	14.41	13.27	12.15
.5	19.69	19.18	18.51	17.48	16.21	14.97
.6	22.64	22.14	21.43	20.36	19.02	17.68
.7	25.35	24.84	24.14	23.05	21.72	20.33
.8	27.85	27.38	26.68	25.63	24.30	22.90
.9	30.15	29.60	29.07	28.05	26.75	25.40
1.0	32.30	31.86	31.29	30.35	29.12	27.82
1.2	36.16	35.83	35.34	34.60	33.58	32.49
1.4	39.52	39.30	38.96	38.44	37.71	36.98
1.6	42.50	42.39	42.21	41.92	41.53	41.09
1.8	45.14	45.13	45.12	45.11	45.09	45.07
2.0	47.55	48.17	47.78	48.03	48.40	48.84
2.5	52.42	52.72	53.39	54.36	55.79	57.53
3.0	56.33	56.93	57.97	59.59	62.11	65.27
3.5	59.51	60.34	61.74	64.02	67.55	72.20
4.0	62.10	63.18	64.82	67.74	72.33	78.45
5.0	66.17	67.45	69.78	73.81	80.27	89.28
6.0	69.20	70.94	73.60	78.49	86.61	98.35
7.0	71.53	73.46	76.57	82.24	91.77	105.98
8.0	73.39	75.67	78.95	85.26	96.10	112.57
9.0	74.89	77.20	80.90	87.77	99.72	118.34
10.0	76.15	78.60	82.55	89.91	102.84	123.31
12.0	78.11	80.78	85.13	93.30	107.89	131.71
14.0	79.58	82.45	86.62	95.89	111.83	138.39
20.0	82.37	85.58	90.82	100.91	120.50	152.36
30.0	84.67	88.17	93.95	105.21	126.57	165.33

Table XIII.

$n = .050$.

$\sqrt{r}$	Slopes. .001	.0004	.0002	.0001	.00005	.000025
.1	3.63	3.48	3.31	3.04	2.74	2.47
.2	6.66	6.45	6.16	5.75	5.26	4.81
.3	9.59	9.32	8.94	8.39	7.74	7.13
.4	12.31	11.98	11.54	10.91	10.14	9.39
.5	14.83	14.47	13.98	13.16	12.43	11.59
.6	17.15	16.79	16.29	15.55	14.64	13.75
.7	19.33	19.02	18.45	17.71	16.79	15.85
.8	21.35	21.00	20.51	19.76	18.85	17.90
.9	23.25	22.92	22.45	21.73	20.84	19.90
1.0	25.04	24.73	24.30	23.61	22.75	21.86
1.2	28.28	28.04	27.69	27.14	26.40	25.51
1.4	31.18	30.99	30.75	30.35	29.83	28.49
1.6	33.76	33.67	33.54	33.33	33.04	32.74
1.8	36.10	36.09	36.08	36.07	36.05	36.04
2.0	38.21	38.30	38.43	38.64	38.88	39.22
2.5	42.65	43.02	43.49	44.16	45.32	46.62
3.0	46.34	46.86	47.67	49.00	50.93	53.32
3.5	49.38	50.14	51.17	53.22	55.90	59.44
4.0	52.48	52.77	54.18	56.60	60.28	65.02
5.0	55.88	57.09	59.02	62.41	67.67	74.82
6.0	58.83	60.38	62.77	67.00	73.83	83.32
7.0	61.22	62.97	65.73	70.70	78.93	90.58
8.0	63.21	65.06	68.16	73.77	83.21	96.92
9.0	64.62	66.80	70.18	76.35	86.90	102.52
10.0	65.99	68.26	71.87	78.54	90.02	107.52
12.0	68.01	70.55	74.58	82.11	95.22	115.94
14.0	69.54	72.29	76.64	84.77	99.33	122.70
20.0	72.58	75.65	80.68	90.21	107.78	137.31
30.0	75.09	78.48	84.07	94.91	115.24	151.43

Table XIV.

Table of numbers and their square roots, used principally for r and s—from .000005 to 100.0.

x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$
.000005	.00236	.0070	.08367	1.00	1.0000
.000010	.00316	.0075	.08660	1.05	1.0247
.000015	.00387	.0080	.08944	1.10	1.0488
.000020	.00472	.0085	.09220	1.15	1.0724
.000025	.00500	.0090	.09487	1.20	1.0954
.000030	.00548	.0095	.09447	1.25	1.1180
.000035	.00592	.016	.1	1.30	1.1402
.000040	.00632	.015	.1225	1.35	1.1619
.000045	.00671	.020	.1414	1.40	1.1832
.000050	.00707	.025	.1581	1.45	1.2042
.000055	.00742	.030	.1732	1.50	1.2247
.000060	.00775	.035	.1871	1.55	1.2449
.000065	.00806	.040	.2	1.60	1.2649
.000070	.00837	.045	.2121	1.65	1.2845
.000075	.00866	.050	.2236	1.70	1.3038
.000080	.00894	.055	.2345	1.75	1.3229
.000085	.00922	.060	.2449	1.80	1.3416
.000090	.00949	.065	.255	1.85	1.3601
.000095	.00975	.070	.2646	1.90	1.3784
......	.....	.075	.2739	1.95	1.3964
.0001	.01000	.080	.2828	2.00	1.4142
.0002	.01414	.085	.2915	2.05	1.4318
.0003	.01732	.090	.3	2.10	1.4491
.0004	.02000	.095	.3082	2.15	1.4663
.0005	.02236	.10	.3162	2.20	1.4832
.0006	.02449	.15	.3872	2.25	1.5000
.0007	.02646	.20	.4472	2.30	1.5166
.0008	.02828	.25	.5	2.35	1.5330
.0009	.03	.30	.5477	2.40	1.5492
.0010	.03162	.35	.5916	2.45	1.5652
.0015	.03873	.40	.6324	2.50	1.5811
.0020	.04472	.45	.6708	2.55	1.5969
......	.....	.50	.7071	2.60	1.6125
.0025	.05	.55	.7416	2.65	1.6279
.0030	.05477	.60	.7746	2.70	1.6432
.0035	.05916	.65	.8062	2.75	1.6583
.0040	.06325	.70	.8367	2.80	1.6733
.0045	.06708	.75	.8660	2.85	1.6882
.0050	.07071	.80	.8944	2.90	1.7029
.0055	.07416	.85	.9220	2.95	1.7176
.0060	.07746	.90	.9487	3.00	1.7321
.0065	.08062	.95	.9747	3.10	1.7607

Table XIV—Continued.

Table of numbers and their square roots, used principally for r and s—from .000005 to 100.0.

x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$
3.2	1.7888	7.3	2.7019	11.4	3.3764
3.3	1.8166	7.4	2.7203	11.5	3.3912
3.4	1.8439	7.5	2.7386	11.6	3.4059
3.5	1,8708	7.6	2.7568	11.7	3.4205
3.6	1.8974	7.7	2.7749	11.8	3.4351
3.7	1.9235	7.8	2.7928	11.9	3.4496
3.8	1.9494	7.9	2.8107	12.0	3.4641
3.9	1.9748	8.0	2.8284	12.1	3.4785
4.0	2.0000	8.1	2.8460	12.2	3.4928
4.1	2.0248	8.2	2.8636	12.3	3.5071
4.2	2.0494	8.3	2.8810	12.4	3.5214
4.3	2.0736	8.4	2.8983	12.5	3.5355
4.4	2.0976	8.5	2.9155	12.6	3.5497
4.5	2.1213	8.6	2.9326	12.7	3.5637
4.6	2.1448	8.7	2.9496	12.8	3.5777
4.7	2.1679	8.8	2.9665	12.9	3.5917
4.8	2.1909	8.9	2.9833	13.0	3.6056
4.9	2.2136	9.0	3.0000	13.1	3.6194
5.0	2.2361	9.1	3.0166	13.2	3.6332
5.1	2.2583	9.2	3.0332	13.3	3.6469
5.2	2.2804	9.3	3.0496	13.4	3.6606
5.3	2.3022	9.4	3.0659	13.5	3.6742
5.4	2.3238	9.5	3.0822	13.6	3.6878
5.5	2.3452	9.6	3.0984	13.7	3.7014
5.6	2.3664	9.7	3.1145	13.8	3.7148
5.7	2.3875	9.8	3.1305	13.9	3.7283
5.8	2.4083	9.9	3.1464	14.0	3.7417
5.9	2.4290	10.0	3.1623	14.1	3.7550
6.0	2.4495	10.1	3.1780	14.2	3.7683
6.1	2.4698	10.2	3.1937	14.3	3.7815
6.2	2.4900	10.3	3.2094	14.4	3.7947
6.3	2.5100	10.4	3.2249	14.5	3.8079
6.4	2.5298	10.5	3.2404	14.6	3.8210
6.5	2.5495	10.6	3.2558	14.7	3.8341
6.6	2.5690	10.7	3.2711	14.8	3.8471
6.7	2.5884	10.8	3.2863	14.9	3.8601
6.8	2.6077	10.9	3.3015	15.0	3.8730
6.9	2.6268	11.0	3.3166	15.1	3.8859
7.0	2.6458	11.1	3.3317	15.2	3.8987
7.1	2.6646	11.2	3.3466	15.3	3.9115
7.2	2.6833	11.3	3.3615	15.4	3.9243

Table XIV—Continued.

Table of numbers and their square roots, used principally for *r* and *s*—from .000005 to 100.0.

x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$
15.5	3.9370	19.6	4.4272	37	6.0828
15.6	3.9497	19.7	4.4384	38	6.1644
15.7	3.9623	19.8	4.4497	39	6.2450
15.8	3.9749	19.9	4.4609	40	6.3246
15.9	3.9875	20.0	4.4721	41	6.4031
16.0	4.0000	20.2	4.4944	42	6.4807
16.1	4.0125	20.4	4.5166	43	6.5574
16.2	4.0249	20.6	4.5387	44	6.6332
16.3	4.0373	20.8	4.5607	45	6.7082
16.4	4.0497	21.0	4.5826	46	6.7223
16.5	4.0613	21.2	4.6043	47	6.8557
16.6	4.0743	21.4	4.6260	48	6.9282
16.7	4.0865	21.6	4.6476	49	7.0000
16.8	4.0988	21.8	4.6691	50	7.0711
16.9	4.1109	22.0	4.6904	51	7.1414
17.0	4.1231	22.2	4.7117	52	7.2111
17.1	4.1352	22.4	4.7328	53	7.2801
17.2	4.1473	22.6	4.7539	54	7.3485
17.3	4.1593	22.8	4.7749	55	7.4162
17.4	4.1713	23.0	4.7958	56	7.4833
17.5	4.1833	23.2	4.8167	57	7.5498
17.6	4.1952	23.4	4.8373	58	7.6158
17.7	4.2071	23.6	4.8579	59	7.6811
17.8	4.2190	23.8	4.8785	60	7.7460
17.9	4.2308	24.0	4.8989	61	7.8102
18.0	4.2427	24.2	4.9194	62	7.8740
18.1	4.2544	24.4	4.9396	63	7.9373
18.2	4.2662	24.6	4.9598	64	8.0000
18.3	4.2779	24.8	4.9800	65	8.0623
18.4	4.2895	25.0	5.0000	66	8.1240
18.5	4.3012	26	5.0990	67	8.1854
18.6	4.3127	27	5.1961	68	8.2462
18.7	4.3229	28	5.2915	69	8.3066
18.8	4.3358	29	5.3852	70	8.3666
18.9	4.3474	30	5.4772	71	8.4261
19.0	4.3590	31	5.5678	72	8.4853
19.1	4.3704	32	5.6569	73	8.5440
19.2	4.3817	33	5.7446	74	8.6023
19.3	4.3932	34	5.8309	75	8.6602
19.4	4.4046	35	5.9161	76	8.7178
19.5	4.4158	36	6.0000	77	8.7750

Table XIV—Continued.

Table of numbers and their square roots, used principally for *r* and *s*—from .000005 to 100.0.

x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$
78	8.8318	86	9.2736	94	9.6954
79	8.8882	87	9.3274	95	9.7468
80	8.9443	88	9.3808	96	9.7980
81	9.0000	89	9.4340	97	9.8489
82	9.0554	90	9.4868	98	9.8995
83	9.1104	91	9.5394	99	9.9499
84	9.1652	92	9.5917	100	10.0000
85	9.2195	93	9.6437		

Table XV.

Table of areas, hydraulic radii, wetted perimeters and central angles for circular segments (diameter of circle = 1).

Versed Sine	Area Constant	Hydraulic Rad. Const.	Wetted Perimeter Constant.	Central Angle
.01	.00133	.0144	.0931	10°41′
.02	.00375	.0131	.28521	32°40′
.03	.00686	.0195	.35007	40° 6′
.04	.01054	.0256	.41271	46° 8′
.05	.01468	.0325	.4513	51°42′
.06	.01924	.0388	.4950	56°42′
.07	.02417	.0451	.53576	61°22′
.08	.02943	.0513	.57382	65°44′
.09	.03501	.0574	.60961	69°50′
.10	.04087	.0635	.6437	73°44′
.12	.05338	.0754	.70774	81° 4′
.14	.06683	.0882	.75737	87°54′
.1464	.07131	.09079	.7854	90° 0′
.16	.08111	.0986	.82324	94°18′
.18	.09613	.1096	.87650	100°24′
.20	.11182	.1206	.92739	106°16′
.22	.12811	.1311	.97680	111°54′
.24	.14494	.1415	1.0243	117°20′
.25	.15360	.1464	1.0481	119°54′
.26	.16226	.1513	1.0720	122°48′
.28	.18002	.1612	1.1168	127°56′
.30	.19817	.1708	1.1596	132°50′
.35	.24498	.1933	1.2667	145° 6′
.40	.29337	.1996	1.4691	156°54′
.45	.34278	.2329	1.4713	168°32′
.50	.39270	.2500	1.5708	180° 0′
.55	.4426	.2648	1.6713	191°28′
.60	.4920	.2775	1.7730	203° 6′
.65	.5404	.2934	1.8755	214°54′
.70	.5872	.2962	1.9825	227°10′
.75	.63185	.3015	2.0955	240° 4′
.80	.6738	.3042	2.2150	253°44′
.82	.6893	.3041	2.2660	259°36′
.84	.7043	.3036	2.3196	265°42′
.86	.7186	.3025	2.3755	272° 6′
.88	.7320	.3006	2.4350	278°56′
.90	.7445	.2991	2.4990	286°16′
.91	.7504	.2964	2.5330	290°10′
.92	.7560	.2943	2.5688	294°16′

Table XV.—Continued.

Table of areas, hydraulic radii, wetted perimeters and central angles for circular segments (diameter of circle = 1).

Versed Sine	Area Constant	Hydraulic Rad. Const.	Wetted Perimeter Constant.	Central Angle
.93	.7612	.2918	2.6070	298°38′
.94	.7662	.2893	2.6478	303°18′
.95	.7707	.2864	2.6914	308°18′
.96	.7749	.2829	2.7390	313°52′
.97	.7785	.2787	2.7927	319°54′
.98	.7816	.2735	2.8573	327°20′
.99	.7841	.2571	3.0494	349°19′
1.00	.7854	.2500	3.1416	360° 0′

Rule—To find area of segment multiply square of diameter by area constant.

To find hydraulic radius multiply diameter by hydraulic radius constant.

To find wetted perimeter multiply diameter by wetted perimeter constant.

Table XVI.

Table of areas, hydraulic radii and wetter perimeter for composite section. (Diameter of circle = 1.)

Depth	Area Constant.	Hydraulic Radius Constant.	Wetted Perimeter Constant.	Circular Depth
.01	.0001	.0036	.02828	.....
.02	.0004	.0071	.05656	.....
.03	.0009	.0106	.08484	.....
.04	.0016	.0141	.11312	.....
.05	.0025	.0180	.14140	.....
.06	.0036	.0216	.16968	.....
.07	.0049	.0252	.19796	.....
.08	.0064	.0288	.22624	.....
.09	.0081	.0324	.25752	.....
.10	.0100	.0360	.2828	.....
.12	.0144	.0432	.33936	.....
.14	.0196	.0504	.39592	.....
.16	.0256	.0576	.45248	.....
.18	.0324	.0648	.50904	.....
.20	.0400	.0720	.5656	.....
.22	.0484	.0792	.62216	.....
.24	.0576	.0864	.67872	.....
.26	.0676	.0936	.73528	.....
.28	.0784	.1008	.79182	.....
.30	.0900	.1080	.84840	.....
.35	.1225	.1245	.9898	.....
.3535	.12494	.12494	1.0000	.14645
.367	.13471	.12981	1.0378	.16
.387	.14973	.13723	1.0911	.18
.407	.16542	.14489	1.1420	.20
.427	.18171	.15255	1.1912	.22
.447	.19854	.1602	1.2389	.24
.467	.21586	.1677	1.2866	.26
.487	.23362	.1755	1.3314	.28
.507	.25177	.1833	1.3742	.30
.557	.29858	.2016	1.4813	.35
.607	.34697	.2061	1.6843	.40
.657	.39638	.2351	1.6859	.45
.707	.44630	.2503	1.7854	.50
.757	.4962	.2631	1.8859	.55
.807	.5456	.2743	1.9876	.60
.857	.5940	.2840	2.0901	.65
.907	.6408	.2916	2.1971	.70
.957	.68545	.2968	2.3101	.75

Table XVI.—Continued.

Table of areas, hydraulic radii and wetted perimeter for composite section. (Diameter of circle = 1.)

Depth	Area Constant.	Hydraulic Radius Constant.	Wetted Perimeter Constant.	Circular Depth
1.007	.7274	.2996	2.4296	.80
1.027	.7429	.2995	2.4806	.82
1.047	.7579	.2991	2.5342	.84
1.067	.7722	.2981	2.5901	.86
1.087	.7856	.2965	2.6496	.88
1.107	.7981	.2941	2.7136	.90
1.117	.8040	.2926	2.7476	.91
1.127	.8096	.2908	2,7834	.92
1.137	.8148	.2887	2.8216	.93
1.147	.8198	.2864	2.8624	.94
1.157	.8243	.2835	2.9060	.95
1.167	.8285	.2805	2.9536	.96
1.177	.8321	.2767	3.0073	.97
1.187	.8352	.2719	3.0719	.98
1.197	.8377	.2567	3.2640	.99
1.207	.8390	.2499	3.3562	1.00

Rule—To find area of flow multiply square of diameter with area constant corresponding the depth of flow.

To find hydraulic radius multiply diameter with hydraulic radius constant.

Table XVII.

Table of areas and circumferences of circles from ⅛ to 60 diameter.

Diameter			
Com. Frac.	Dec. Frac.	Circumference	Area
⅛	.125	.3927	.012272
¼	.250	.7854	.049087
⅜	.375	1.1781	.110447
½	.5	1.5708	.19635
⅝	.625	1.9635	.306796
¾	.750	2.3562	.441787
⅞	.875	2.7489	.601322
1	1.000	3.1416	.7854
1⅛	1.125	3.5343	.99402
1¼	1.250	3.927	1.2272
1⅜	1.375	4.3197	1.4849
1½	1.50	4.7124	1.7671
1⅝	1.625	5.1051	2.0739
1¾	1.750	5.4978	2.4053
1⅞	1.875	5.8905	2.7612
2	2.00	6.2832	3.1416
2⅛	2.125	6.6759	3.5466
2¼	2.250	7.0686	3.9761
2⅜	2.375	7.4613	4.4301
2½	2.5	7.854	4.9087
2⅝	2.625	8.2467	5.4119
2¾	2.750	8.6394	5.9396
2⅞	2.875	9.031	6.4918
3	3.00	9.4248	7.0686
3⅛	3.125	9.8175	7.6699
3¼	3.250	10.2102	8.2958
3⅜	3.375	10.6029	8.9462
3½	3.5	10.9956	9.6211
3⅝	3.625	11.3883	10.3206
3¾	3.75	11.781	11.0447
3⅞	3.875	12.1737	11.7933
4	4.0	12.5664	12.5664
4⅛	4.125	12.9591	13.3641
4¼	4.25	13.3518	14.1863
4⅜	4.375	13.7445	15.033
4½	4.5	14.1372	15.9043
4⅝	4.625	14.5299	16.8002
4¾	4.75	14.9226	17.7206
4⅞	4.875	15.3153	18.6555
5	5.0	15.708	19.635

Table XVII.—Continued.

Table of areas and circumferences of circles from ⅛ to 60 diameter.

Diameter			
Com. Frac.	Dec. Frac.	Circumference	Area
5⅛	5.125	16.1007	20.629
5¼	5.25	16.4934	21.6476
5⅜	5.375	16.8861	22.6907
5½	5.50	17.2788	23.7583
5⅝	5.625	17.6717	24,8505
5¾	5.75	18.0642	25.9673
5⅞	5.875	18.4569	27.1086
6	6.0	18.8496	28.2744
6¼	6.25	19.635	30.6797
6½	6.5	20.4204	33.1831
6¾	6.75	21.2058	35.7848
7	7.0	21.9912	38.4846
7¼	7.25	22.7766	41.2826
7½	7.5	23.562	44.1787
7¾	7.75	24.3474	47.1731
8	8.0	25.1328	50.2656
8¼	8.25	25.9182	53.4563
8½	8.50	26.7036	56.7451
8¾	8.75	27.489	60.1322
9	9.0	28.2744	63.6174
9¼	9.25	29.0598	67.2008
9½	9.50	29.8452	70.8823
9¾	9.75	30.6306	74.6621
10	10.0	31.416	78.54
10¼	10.25	32.2014	82.5161
10½	10.50	32.9868	86.5903
10¾	10.75	33.7722	90.7628
11	11.00	34.5576	95.0334
11¼	11.25	35.343	99.4022
11½	11.50	36.1284	103.8691
11¾	11.75	36.9138	108.4343
12	12.0	37.6992	113.098
12¼	12.25	38.4846	117.859
12½	12.50	39.27	122.719
12¾	12.75	40.0554	127.677
13	13.0	40.8408	132.733
13¼	13.25	41.6262	137.887
13½	13.50	42.4116	143.139
13¾	13.75	43.197	148.49
14	14.0	43.9824	153.938

Table XVII.—Continued.

Table of areas and circumferences of circles from ⅛ to 60 diameter.

Diameter			
Com. Frac.	Dec. Frac.	Circumference	Area
14¼	14.25	44.7678	159.485
14½	14.50	45.5532	165.13
14¾	14.75	46.3386	170.874
15	15.0	47.124	176.715
15¼	15.25	47.9094	182.655
15½	15.50	48.6949	188.692
15¾	15.75	49.4802	194.828
16	16.0	50.2656	201.062
16¼	16.25	51.051	207.395
16½	16.50	51.8364	213.825
16¾	16.75	52.6218	220.354
17	17.0	53.4072	226.931
17¼	17.25	54.1926	233.706
17½	17.50	54.978	240.529
17¾	17.75	55.7634	247.45
18	18.0	56.5488	254.47
18¼	18.25	57.3342	261.587
18½	18.50	58.1196	268.803
18¾	18.75	58.905	276.117
19	19.0	59.6904	283.529
19¼	19.25	60.4758	291.04
19½	19.50	61.2612	298.648
19¾	19.75	62.0466	306.355
20	20.0	62.832	314.16
20¼	20.25	63.6174	322.063
20½	20.50	64.4028	330.064
20¾	19.75	65.1882	338.164
21	21.0	65.9736	346.361
21¼	21.25	66.759	354.657
21½	21.50	67.5444	363.051
21¾	21.75	68.3298	371.543
22	22.0	69.1152	380.134
22¼	22.25	69.9006	388.822
22½	22.50	70.686	397.609
22¾	22.75	71.4714	406.494
23	23.0	72.2568	415.477
23¼	23.25	73.0422	425.558
23½	23.50	73.8276	433.737
23¾	23.75	74.613	443.737
24	24.0	75.3984	452.39
24½	24.50	76.9692	471.436

Table XVII.—Continued.

Table of areas and circumferences of circles from ⅛ to 60 diameter.

Diameter			
Com. Frac.	Dec. Frac.	Circumference	Area
25	25.0	78.54	490.875
25½	25.50	80.1108	510.706
26	26.0	81.6816	530.93
26½	26.50	83.2524	551.547
27	27.	84.8232	572.557
27½	27.50	86.394	593.959
28	28.	87.9648	615.754
28½	28.50	89.5356	637.941
29	29.0	91.1064	660.521
29½	29.5	92.6772	683.494
30	30.0	94.248	706.86
31	....	97.3896	754.769
32	....	100.5312	804.25
33	....	103.673	855.301
34	....	106.814	907.922
35	....	109.956	962.115
36	....	113.098	1017.878
37	....	116.239	1075.213
38	....	119.381	1134.118
39	....	122.522	1194.593
40	....	125.664	1256.64
41	....	128.806	1320.26
42	....	131.947	1385.45
43	....	135.089	1452.2
44	....	138.23	1520.53
45	....	141.372	1590.43
46	....	144.514	1661.91
47	....	147.655	1734.95
48	....	150.797	1809.56
49	....	153.938	1885.75
50	....	157.08	1963.5
51	....	160.222	2042.83
52	....	163.363	2123.72
53	....	166.505	2206.19
54	....	169.646	2290.23
55	....	172.788	2375.83
56	....	175.93	2463.01
57	....	179.071	2551.76
58	....	182.213	2642.09
59	....	185.354	2733.98
60	....	188.496	2827.44

XVIII.

Table of Hydraulic Equivalents.

1 acre-foot equals 43,560 cubic feet.
1 second-foot equals 40 California miner's inches.
1 second-foot equals 38.4 Colorado miner's inches.
1 second-foot equals 40 Arizona miner's inches.
1 second-foot equals 7.48 United States gallons per second.
1 second-foot equals 448.8 gallons per minute.
1 second-foot equals 646,272 gallons for one day.
1 second-foot equals 6.23 British imperial gallons per second.
1 second-foot for one year covers 1 square mile 1.131 feet or 13.572 inches deep.
1 second-foot for one year equals 31,536,000 cubic feet.
1 second-foot equals about 1 acre-inch per hour.
1 second-foot for one day covers 1 square mile 0.03719 inch deep.
1 second-foot for one day equals 1.983 acre-feet.
100 California miner's inches equal 18.7 United States gallons per second.
100 California miner's inches for one day equal 4.96 acre-feet.
100 Colorado miner's inches equal 2.60 second-feet.
100 Colorado miner's inches equal 19.5 United States gallons per second.
100 Colorado miner's inches equal 104 California miner's inches.
100 Colorado miner's inches for one day equal 5.17 acre-feet.
100 United States gallons per minute equal 0.223 second-foot.
100 United States gallons per minute for one day equal 0.442 acre-foot.
1,000,000 United States gallons per day equal 1.55 second-feet.
1,000,000 United States gallons equal 3.07 acre-feet.
1,000,000 cubic feet equal 22.95 acre-feet.
1 acre-foot equals 325,850 gallons.
1 inch deep on 1 square mile equals 2,323,200 cubic feet.
1 inch deep on 1 square mile equals 0.0737 second-foot per year.
1 acre equals 0.4047 hectare.
1 acre equals 43,560 square feet.
1 acre equals 209 feet square, nearly
1 square mile equals 2.59 square kilometers.
1 cubic foot equals 7.48 gallons.
1 cubic foot of water weighs 62.5 pounds.
1 cubic meter per minute equals 0.5886 second-foot.

XIX.

Table of Weights of a Cubic Foot of Various Substances.

	In pounds.
Anthracite coal, solid, of Pennsylvania	93
Anthracite coal, broken, loose	54
Anthracite coal, moderately shaken	58
Anthracite coal, heaped bushel, loose	80
Ash, American white, dry	38
Asphaltum	87
Brass (Copper and Zinc), cast	504
Brass, rolled	524
Brick, best pressed	150
Brick, common hard	125
Brick, soft, inferior	100
Brickwork, pressed brick	140
Brickwork, ordinary	112
Cement, hydraulic, ground, loose, American, Rosendale	56
Cement, hydraulic, ground, loose, American, Louisville	50
Cement, hydraulic, ground, loose, English, Portland	90
Cherry, dry	42
Chestnut, dry	41
Coal, bituminous, solid	84
Coal, bituminous, broken, loose	49
Coal, bituminous, heaped bushel, loose	74
Coke, loose, of good coal	27
Coke, loose, heaped bushel	38
Copper, cast	542
Copper, rolled	548
Earth, common loam, dry, loose	76
Earth, common loam, dry, moderately rammed	95
Earth, as a soft, flowing mud	108
Ebony, dry	76
Elm, dry	35
Flint	162
Glass, common window	157
Gneiss, common	168
Gold, cast, pure, or 24 carat	1204
Gold, pure, hammered	1217
Granite	170
Gravel, about the same as sand, which see.	
Hemlock, dry	25
Hickory, dry	53
Hornblende, black	203
Ice	58.7
Iron, cast	450

Iron, wrought, purest	485
Iron, wrought, average	480
Ivory	114
Lead	711
Lignum Vitæ, dry	83
Lime, quick, ground, loose, or in small lumps	53
Lime, quick, ground, loose, thoroughly shaken	75
Lime, quick, ground, loose, per struck bushel	66
Limestone and Marbles	168
Limestone and Marbles, loose, in irregular fragments	96
Mahogany, Spanish, dry	53
Mahogany, Honduras, dry	35
Maple, dry	49
Marbles—see Limstone.	
Masonry, of granite or limestone, well dressed	165
Masonry, of mortar rubble	154
Masonry, of dry rubble (well scabbled)	138
Masonry, of sandstone, well dressed	144
Mercury, at 32 degrees Fahrenheit	849
Mica	183
Mortar, hardened	103
Mud, dry, close	80 to 100
Mud, wet fluid, maximum	120
Oak, live, dry	59
Oak, white, dry	52
Oak, other kinds	32 to 45
Petroleum	55
Pine, white, dry	25
Pine, yellow, Northern	34
Pine, yellow, Southern	45
Platinum	1342
Quartz, common, pure	165
Rosin	69
Salt, coarse, Syracuse, N. Y	45
Salt, Liverpool, fine, for table use	49
Sand, of pure quartz, dry, loose	90 to 106
Sand, well shaken	99 to 117
Sand, perfectly wet	120 to 140
Sandstones, fit for building	151
Shales, red or black	162
Silver	655
Slate	175
Snow, freshly fallen	5 to 12
Snow, moistened and compacted by rain	15 to 50
Spruce, dry	25
Steel	490
Sulphur	125

Sycamore, dry	37
Tar	62
Tin, cast	459
Turf or Peat, dry, unpressed	20 to 30
Walnut, black, dry	38
Water, pure rain or distilled, at 60 degrees Fahrenheit	62⅓
Water, sea	64
Wax, bees	60.5
Zinc or Spelter	437

Green timbers usually weigh from one-fifth to one-half more than dry.

XX.

Conversion Table of United States and Metric Measures and Weights.

LINEAR MEASURE.

1/64 of an inch.........is equivalent to .39688 Millimeter
1 Inch ....is equivalent to 2.54001 Centimeters
1 Foot ....is equivalent to 3.04801 Meters
1 Yard ....is equivalent to 9.14402 Meters
1 Statute Mile...........is equivalent to 1.60935 Kilometers
1 Nautical Mile..........is equivalent to 1.85325 Kilometers

SQUARE MEASURE.

1 Square Inch....is equivalent to 6.45163 Square Centimeters
1 Square Foot...is equivalent to .09290 Square Meter
1 Square Yard...is equivalent to .83613 Square Meter
1 Acre ....is equivalent to .40470 Hectare
1 Square Mile....is equivalent to 2.59000 Square Kilometers

CUBIC MEASURE.

1 Cubic Inch.....is equivalent to 16.38716 Cubic Centimeters
1 Cubic Inch.....is equivalent to .01639 Cubic Decimeter
1 Cubic Foot.....is equivalent to .02832 Cubic Meter
1 Cubic Yard....is equivalent to .76456 Cubic Meter

CAPACITY MEASURE.

1 Lq. Qt. is equiv. to .94636 Liter
1 Gal. ...is equiv. to 3.78543 Liters
1 Gal. .. is equiv. to .00379 Cubic Meter
1 Bu. ...is equiv. to .35239 Hectoliter
1 Fl. Dr. is equiv. to 3.69671 Milliliters or Cubic Centimeters
1 Fl. Oz. is equiv. to 29.57370 Milliliters or Cubic Centimeters

WEIGHTS.

1 Grain ....is equivalent to 64.79892 Milligrams
1 Troy Ounce ....is equivalent to 31.10348 Grams
1 Avoirdupois Ounce ....is equivalent to 28.34953 Grams
1 Avoirdupois Pound ...is equivalent to .45359 Kilogram
1 Net Ton of 2,000 lbs..is equivalent to .90718 Tonne
1 Gross Ton of 2,240 lbs.is equivalent to 1.01605 Tonnes

MISCELLANEOUS.

1 Lb. per Lin. Ft.. is equiv to 1.48816 Kilogs. per Lin. Meter
1 Lb. per Sq. Ft... is equiv. to .07031 Kilogs. per Sq. C'ntim't'r
1 Lb. per Sq. Ft...is equiv. to 4.88241 Kilogs. per Sq. Meter
1 Lb. per Cu. Ft.. is equiv. to 16.01837 Kilogs. per Cu. Meter
1 Ft. Pound......is equiv. to .13826 Kilogrammeter
1 U. S. H. P. ..is equiv. to 1.01387 Metric Horse Power

Table XXI.

Conversion Table of Millions of Gallons Per 24 Hours.

Mill. Gals. in 24 Hours	No. of Gals. per 1 Hr.	No. of Gals. per 1 Min.	No. of Gals. per 1 Sec.	No. of Cu. Ft. per 24 Hrs.	No. of Cu. Ft. per 1 Min.	No. of Cu. Ft. per 1 Sec.
1 mill. gals.	41,666.7	694.44	11.58	133,689.8	92.81	1.55
2 mill. gals.	83,333.3	1,388.88	23.15	267,379.6	185.62	3.09
3 mill. gals.	125,000.0	2,083.33	34.72	401,069.5	278.44	4.64
5 mill. gals.	208,333.3	3,372.20	57.85	668,449.2	464.06	7.74
7 mill. gals.	291,666.7	4,861.08	81.02	935,828.9	649.68	10.83
11 mill. gals.	458,333.4	7,638.84	127.31	1,470,588	1,026.93	17.02
13 mill. gals.	541,666.7	9,027.72	150.46	1,737,968	1,206.6	20.11
17 mill. gals.	708,333.2	11,805.5	196.76	2,272,727	1,577.8	26.30
19 mill. gals.	811,666.7	13,194.4	219.91	2,540,107	1,763.4	29.37
23 mill. gals.	958,333.3	15,972.1	266.20	3,074,866	2,134.7	35.58
30 mill. gals.	1,250,000	20,833.2	347.22	4,010,695	2,784.4	46.41
40 mill. gals.	1,666,667	27,777.6	462.96	5,347,594	3,712.4	61.88
50 mill. gals.	2,083,333	33,722.0	578.50	6,684,492	4,640.6	77.35
60 mill. gals.	2,500,000	41,666.4	694.44	8,021,390	5,568.7	92.82
70 mill. gals.	2,916,667	48,610.8	810.18	9,358,289	6,496.8	108.29
80 mill. gals.	3,333,333	55,555.2	925.92	10,695,197	7,424.7	123.76
90 mill. gals.	3,756,000	82,499.6	1,041.66	12,032,056	8,353.1	139.23
100 mill. gals.	4,166,667	67,444.0	1,157.00	13,368,984	9,281.2	154.7

Mill. Gals. in 24 Hours	Lbs. of Water in 24 Hrs.	Lbs. of Water in 1 Min.	Lbs. of Water in 1 Sec.	Water H.P. for 1 Ft. Head	Flow Area if v=1 ft.	Diam. of Pipe if v=1 ft.
1 mill. gals.	8,355,615	5,802.5	96.71	.176	1.547	16.85"
2 mill. gals.	16,711,230	11,605.0	193.42	.352	3.094	23.98"
3 mill. gals.	25,066,845	17,408	290.13	.528	4.641	29.13"
5 mill. gals.	41,778,075	29,013	483.55	.880	7.735	37.65"
7 mill. gals.	58,489,305	40,617	677.0	1.232	10.829	44.6 "
11 mill. gals.	91,911,765	63,827	1,063.8	1.936	17.017	55.85"
13 mill. gals.	108,622,995	75,430	1,257.2	2.288	20.111	60.72"
17 mill. gals.	142,045,455	98,643	1,644.1	2.992	26.299	69.45"
19 mill. gals.	158,756,685	110,248	1,837.5	3.344	29.373	73.4 "
23 mill. gals	192,179,145	133,458	2,224.3	4.044	35.581	80.8 "
30 mill. gals.	250,668,450	174,075	2,901.3	5.280	46.41	92.26"
40 mill. gals.	334,224,600	232,100	3,868.4	7.04	61.88	106.56"
50 mill. gals.	417,780,750	290,125	4,835.5	8.80	77.35	119.04"
60 mill. gals.	501,336,900	348,150	5,802.6	10.56	92.82	130.56"
70 mill. gals.	584,893,050	406,175	6,769.7	12.32	108.29	140.88"
80 mill. gals.	668,449,200	464,200	7,736.8	14.08	123.76	150.72"
90 mill. gals.	752,005,350	522,225	8,703.9	15.84	139.23	159.84"
100 mill. gals.	835,561,500	580,250	9,671.0	17.60	154.70	168.48"

Application—What is the water horse-power of a pump lifting 5 million gallons of water 42 ft. high in 24 hours?

Solution—Under column headed "water horse-power for 1 ft. of head" find number opposite 5 mill. gals., which is .880; multiply this by the lift, 42 × .88 = 39.96 net water horse-power; for actual requirements double the net horse-power, which gives 73.92, or, say 75 horse-power in round numbers, will be required for the above problem. This, of course, means for continuous pumping for 24 hours; if the 5 million gallons is to be pumped in 12 hours the required horse-power is to be doubled.

As one acre contains 43,560 sq. ft., the depth of water in feet is found by dividing the given quantity in cubic feet by 43,560, for instance: the depth of water due to 11 million gallons in 24 hours equals 1,470,580 ÷ 43,560 = 33.76 ft.; if this water is distributed over 160 acres it would cover that area to a depth of 12 × 33.76 ÷ 160 = 2.532 in. or 2 17/32 in. deep.

Table XXII.

Sizes of pipes or cylindrical conduits required for the flow of given quantities of water at given velocities:

Mill. Gals. 24 Hrs.	v=1 fs. Dia. in.	v=2 fs. Dia. in.	v=3 fs. Dia. in.	v=4 fs. Dia. in.	v=5 fs. Dia. in.	v=6 fs. Dia. in.
1	16.85	11.9	9.8	8.4	7.5	6.9
2	23.98	16.85	13.75	11.9	10.65	9.8
3	29.13	20.64	16.85	14.6	13.25	11.9
5	37.65	26.65	21.75	18.85	16.85	15.35
7	44.6	31.5	25.75	22.3	19.9	18.2
11	55.85	39.6	32.25	27.95	25.0	22.8
13	60.72	42.95	35.10	30.36	27.15	24.76
17	69.45	49.1	40.14	34.73	31.06	28.34
19	73.4	51.9	42.43	36.7	32.84	29.94
23	80.8	57.1	46.72	40.4	36.13	32.92
30	92.26	65.25	53.27	46.13	41.26	37.65
40	106.56	75.37	61.53	53.28	47.66	43.49
50	119.04	84.20	68.75	59.52	53.24	48.58
60	130.56	89.36	75.39	65.28	58.39	53.28
70	140.88	99.66	81.35	70.44	62.97	57.49
80	150.72	106.59	87.03	75.36	67.40	61.51
90	159.84	113.04	92.29	79.92	71.49	65.23
100	168.48	119.88	97.29	84.24	75.35	68.75

Mill. Gals. 24 Hrs.	v=7 fs. Dia. in.	v=8 fs. Dia. in.	v=9 fs. Dia. in.	v=10 fs. Dia. in.	v=11 fs. Dia. in.	v=12 fs. Dia. in.
1	6.35	6.0	5.6	5.35	5.05	4.85
2	8.9	8.4	7.95	7.55	7.18	6.87
3	11.0	10.3	9.7	9.2	8.8	8.4
5	14.36	13.33	12.55	11.90	11.35	10.87
7	16.85	15.75	14.85	14.10	13.45	12.87
11	21.11	19.75	18.65	17.67	16.84	16.14
13	22.94	21.46	20.24	19.21	18.60	17.55
17	26.23	24.55	23.15	21.97	20.95	20.06
19	27.72	25.96	24.46	23.20	22.19	21.20
23	30.52	28.57	26.93	25.53	24.39	23.31
30	34.86	32.62	30.75	29.19	27.81	26.63
40	40.58	37.68	35.52	33.71	32.12	30.75
50	44.96	42.09	39.68	37.67	35.90	34.37
60	49.32	46.16	43.52	41.32	39.36	37.69
70	53.23	49.82	46.96	44.58	42.49	40.67
80	56.93	53.29	50.24	47.69	45.44	43.51
90	60.40	56.52	53.28	50.59	48.18	46.16
100	63.61	59.51	56.16	53.31	50.79	48.64

Application—What size pipe is required for the flow of 23 mill. gallons of water in 24 hours at a mean velocity of 6 ft. per second?

Answer—Look in first column, follow line 23 horizontally until column headed v = 6fs.; in this column the diameter is given as 32.92 inches, or nearly 33 inches. The nearest commercial size to be taken then would be 36 in., which would accommodate that flow with slightly smaller velocity.

For quantities not shown, find size by interpolation; for instance: Find size of conduit required for the flow of 55 millions of gallons in 24 hours at a velocity of 4fs.

Answer—For 50 million gallons the required diameter in column 4fs. is 59.52.

For 60 million gallons the required diameter in column 4fs. is 65.28.

Add the two diameters together = 124.80 and divide by 2 gives 62.4 inches as the diameter required. A conduit 5 ft. in diameter would do the work with a slightly increased velocity.

SQUARES CUBES, SQUARE AND CUBE ROOTS.

NUMBER.	SQUARE.	CUBE.	SQUARE ROOT.	CUBE ROOT.
1	1	1	1	1
2	4	8	1.414 213 6	1.259 921
3	9	27	1.732 050 8	1.442 249 6
4	16	64	2	1.587 401 1
5	25	125	2.236 068	1.709 975 9
6	36	216	2.449 489 7	1.817 120 6
7	49	343	2.645 751 3	1.912 931 2
8	64	512	2.828 427 1	2
9	81	729	3	2.080 083 7
10	1 00	1 000	3.162 277 7	2.154 434 7
11	1 21	1 331	3.316 624 8	2.223 980 1
12	1 44	1 728	3.464 101 6	2.289 428 6
13	1 69	2 197	3.605 551 3	2.351 334 7
14	1 96	2 744	3.741 657 4	2.410 142 2
15	2 25	3 375	3.872 983 3	2.466 212 1
16	2 56	4 096	4	2.519 842 1
17	2 89	4 913	4.123 105 6	2.571 281 6
18	3 24	5 832	4.242 640 7	2,620 741 4
19	3 61	6 859	4.358 598 9	2 668 401 6
20	4 00	8 000	4.472 136	2.714 417 7
21	4 41	9 261	4.582 575 7	2.758 924 3
22	4 84	10 648	4.690 415 8	2.802 039 3
23	5 29	12 167	4.795 831 5	2.843 867
24	5 76	13 824	4.898 979 5	2.884 499 1
25	6 25	15 625	5	2.924 017 7
26	6 76	17 576	5.099 019 5	2.962 496
27	7 29	19 683	5.196 152 4	3
28	7 84	21 952	5 291 502 6	3.036 588 9
29	8 41	24 389	5.385 164 8	3.072 316 8
30	9 00	27 000	5.477 225 6	3 107 232 5
31	9 61	29 791	5.567 764 4	3.141 380 6
32	10 24	32 768	5.656 854 2	3.174 802 1
33	10 89	35 937	5 744 562 6	3.207 534 3
34	11 56	39 304	5.830 951 9	3.239 611 8
35	12 25	42 875	5.916 079 8	3.271 066 3
36	12 96	46 656	6	3.301 927 2
37	13 69	50 653	6.082 762 5	3.332 221 8
38	14 44	54 872	6.164 414	3.361 975 4
39	15 21	59 319	6.244 998	3.391 211 4
40	16 00	64 000	6.324 555 3	3.419 951 9
41	16 81	68 921	6.403 124 2	3.448 217 2
42	17 64	74 088	6.480 740 7	3 476 026 6
43	18 49	79 507	6.557 438 5	3.503 398 1
44	19 36	85 184	6.633 249 6	3.530 348 3
45	20 25	91 125	6.708 203 9	3.556 893 3
46	21 16	97 336	6.782 33	3.583 047 9
47	22 09	103 823	6.855 654 6	3.608 826 1
48	23 04	110 592	6.928 203 2	3.634 241 1
49	24 01	117 649	7	3.659 305 7
50	25 00	125 000	7.071 067 8	3.684 031 4
51	26 01	132 651	7.141 428 4	3.708 429 8
52	27 04	140 608	7.211 102 6	3.732 511 1
53	28 09	148 877	7.280 109 9	3.756 285 8
54	29 16	157 464	7.348 469 2	3.779 763 1

NUMBER.	SQUARE.	CUBE.	SQUARE ROOT.	CUBE ROOT.
55	30 25	166 375	7.416 198 5	3.802 952 5
56	31 36	175 616	7.483 314 8	3.825 862 4
57	32 49	185 193	7.549 834 4	3.848 501 1
58	33 64	195 112	7.615 773 1	3.870 876 6
59	34 81	205 379	7.681 145 7	3.892 996 5
60	36 00	216 000	7.745 966 7	3.914 867 6
61	37 21	226 981	7.810 249 7	3.936 497 2
62	38 44	238 328	7.874 007 9	3.957 891 5
63	39 69	250 047	7.937 253 9	3.979 057 1
64	40 96	262 144	8	4
65	42 25	274 625	8.062 257 7	4.020 725 6
66	43 56	287 496	8.124 038 4	4.041 240 1
67	44 89	300 763	8.185 352 8	4.061 548
68	46 24	314 432	8.246 211 3	4.081 655 1
69	47 61	328 509	8,306 623 9	4.101 566 1
70	49 00	343 000	8,366 600 3	4.121 285 3
71	50 41	357 911	8.426 149 8	4.140 817 8
72	51 84	373 248	8.485 281 4	4.160 167 6
73	53 29	389 017	8.544 003 7	4.179 339
74	54 76	405 224	8.602 325 3	4.198 336 4
75	56 25	421 875	8.660 254	4.217 163 3
76	57 76	438 976	8.717 797 9	4.235 823 6
77	59 29	456 533	8.774 964 4	4.254 321
78	60 84	474 552	8.831 760 9	4.272 658 6
79	62 41	493 039	8.888 194 4	4.290 840 4
80	64 00	512 000	8.944 271 9	4.308 869 5
81	65 61	531 441	9	4.326 748 7
82	67 24	551 368	9.055 385 1	4.344 481 5
83	68 89	571 787	9.110 433 6	4.362 070 7
84	70 56	592 704	9.165 151 4	4.379 519 1
85	72 25	614 125	9.219 544 5	4.396 829 6
86	73 96	636 056	9.273 618 5	4.414 004 9
87	75 69	658 503	9.327 379 1	4.431 047 6
88	77 44	681 472	9.380 831 5	4.447 960 2
89	79 21	704 969	9.433 981 1	4.464 745 1
90	81 00	729 000	9.486 833	4.481 404 7
91	82 81	753 571	9.539 392	4.497 941 4
92	84 64	778 688	9.591 663	4.514 357 4
93	86 49	804 357	9.643 650 8	4.530 654 9
94	88 36	830 584	9.695 359 7	4.546 835 9
95	90 25	857 375	9.746 794 3	4.562 902 6
96	92 16	884 736	9.797 959	4.578 857
97	94 09	912 673	9.848 857 8	4.594 700 9
98	96 04	941 192	9.899 494 9	4.610 436 3
99	98 01	970 299	9 949 874 4	4.626 065
100	1 00 00	1 000 000	10	4.641 588 8
101	1 02 01	1 030 301	10.049 875 6	4.657 009 5
102	1 04 04	1 061 208	10.099 504 9	4.672 328 7
103	1 06 09	1 092 727	10.148 891 6	4.687 548 2
104	1 08 16	1 124 864	10 198 039	4.702 669 4
105	1 10 25	1 157 625	10.246 950 8	4.717 694
106	1 12 36	1 191 016	10.295 630 1	4.732 623 5
107	1 14 49	1 225 043	10 344 080 4	4.747 459 4
108	1 16 64	1 259 712	10.392 304 8	4.762 203 2
109	1 18 81	1 295 029	10.440 306 5	4.776 856 2
110	1 21 00	1 331 000	10.488 088 5	4.791 419 9

NUMBER.	SQUARE.	CUBE.	SQUARE ROOT.	CUBE ROOT.
111	1 23 21	1 367 631	10.535 653 8	4.805 895 5
112	1 25 44	1 404 928	10.583 005 2	4.820 284 5
113	1 27 69	1 442 897	10.630 145 8	4.834 588 1
114	1 29 96	1 481 544	10.677 078 3	4.848 807 6
115	1 32 25	1 520 875	10.723 805 3	4.862 944 2
116	1 34 56	1 560 896	10.770 329 6	4.876 999
117	1 36 89	1 601 613	10.816 653 8	4.890 973 2
118	1 39 24	1 643 032	10.862 780 5	4.904 868 1
119	1 41 61	1 685 159	10.908 712 1	4.918 684 7
120	1 44 00	1 728 000	10.954 451 2	4.932 424 2
121	1 46 41	1 771 561	11	4.946 087 4
122	1 48 84	1 815 848	11.045 361	4.959 675 7
123	1 51 29	1 860 867	11.090 536 5	4.973 189 8
124	1 53 76	1 906 624	11.135 528 7	4.986 631
125	1 56 25	1 953 125	11.180 339 9	5
126	1 58 76	2 000 376	11.224 972 2	5.013 297 9
127	1 61 29	2 048 383	11.269 427 7	5.026 525 7
128	1 63 84	2 097 152	11.313 708 5	5.039 684 2
129	1 66 41	2 146 689	11.357 816 7	5.052 774 3
130	1 69 00	2 197 000	11.401 754 3	5.065 797
131	1 71 61	2 248 091	11.445 523 1	5.078 753 1
132	1 74 24	2 299 968	11.489 125 3	5.091 643 4
133	1 76 89	2 352 637	11.532 562 6	5.104 468 7
134	1 79 56	2 406 104	11.575 836 9	5.117 229 9
135	1 82 25	2 460 375	11.618 95	5.129 927 8
136	1 84 96	2 515 456	11.661 903 8	5.142 563 2
137	1 87 69	2 571 353	11.704 699 9	5.155 136 7
138	1 90 44	2 628 072	11.747 340 1	5.167 649 3
139	1 93 21	2 685 619	11.789 826 1	5.180 101 5
140	1 96 00	2 744 000	11.832 159 6	5.192 494 1
141	1 98 81	2 803 221	11.874 342 1	5.204 827 9
142	2 01 64	2 863 288	11.916 375 3	5.217 103 4
143	2 04 49	2 924 207	11.958 260 7	5.229 321 5
144	2 07 36	2 985 984	12	5.241 482 8
145	2 10 25	3 048 625	12.041 594 6	5.253 587 9
146	2 13 16	3 112 136	12.083 046	5.265 637 4
147	2 16 09	3 176 523	12.124 355 7	5.277 632 1
148	2 19 04	3 241 792	12.165 525 1	5.289 572 5
149	2 22 01	3 307 949	12.206 555 6	5.301 459 2
150	2 25 00	3 375 000	12.247 448 7	5.313 292 8
151	2 28 01	3 442 951	12.288 205 7	5.325 074
152	2 31 04	3 511 808	12.328 828	5.336 803 3
153	2 34 09	3 581 577	12.369 316 9	5.348 481 2
154	2 37 16	3 652 264	12.409 673 6	5.360 108 4
155	2 40 25	3 723 875	12.449 899 6	5.371 685 4
156	2 43 36	3 796 416	12.489 996	5.383 212 6
157	2 46 49	3 869 893	12.529 964 1	5.394 690 7
158	2 49 64	3 944 312	12.569 805 1	5.406 120 2
159	2 52 81	4 019 679	12.609 520 2	5.417 501 5
160	2 56 00	4 096 000	12.649 110 6	5.428 835 2
161	2 59 21	4 173 281	12.688 577 5	5.440 121 8
162	2 62 44	4 251 528	12.727 922 1	5.451 361 8
163	2 65 69	4 330 747	12.767 145 3	5.462 555 6
164	2 68 96	4 410 944	12.806 248 5	5.473 703 7
165	2 72 25	4 492 125	12.845 232 6	5.484 806 6
166	2 75 56	4 574 296	12.884 098 7	5.495 864 7

NUMBER.	SQUARE.	CUBE.	SQUARE ROOT.	CUBE ROOT.
167	27889	4 657 463	12.922 848	5.506 878 4
168	28224	4 741 632	12.961 481 4	5.517 848 4
169	28561	4 826 809	13	5.528 774 8
170	28900	4 913 000	13.038 404 8	5.539 658 3
171	29241	5 000 211	13.076 696 8	5.550 499 1
172	29584	5 088 448	13.114 877	5.561 297 8
173	29929	5 177 717	13.152 946 4	5.572 054 6
174	30276	5 268 024	13.190 906	5.582 770 2
175	30625	5 359 375	13.228 756 6	5.593 444 7
176	30976	5 451 776	13.266 499 2	5.604 078 7
177	31329	5 545 233	13.304 134 7	5.614 672 4
178	31684	5 639 752	13.341 664 1	5.625 226 3
179	32041	5 735 339	13.379 088 2	5.635 740 8
180	32400	5 832 000	13.416 407 9	5.646 216 2
181	32761	5 929 741	13.453 624	5.656 652 8
182	33124	6 028 568	13.490 737 6	5.667 051 1
183	33489	6 128 487	13.527 749 3	5.677 411 4
184	33856	6 229 504	13.564 66	5.687 734
185	34225	6 331 625	13.601 470 5	5.698 019 2
186	34596	6 434 856	13.638 181 7	5.708 267 5
187	34969	6 539 203	13.674 794 3	5.718 479 1
188	35344	6 644 672	13.711 309 2	5.728 654 3
189	35721	6 751 269	13.747 727 1	5.738 793 6
190	36100	6 859 000	13.784 048 8	5.748 897 1
191	36481	6 967 871	13.820 275	5.758 965 2
192	36864	7 077 888	13.856 406 5	5.768 998 2
193	37249	7 189 057	13.892 44	5.778 996 6
194	37636	7 301 384	13.928 388 3	5.788 960 4
195	38025	7 414 875	13.964 24	5.798 89
196	38416	7 529 536	14	5.808 785 7
197	38809	7 645 373	14.035 668 8	5.818 647 9
198	39204	7 762 392	14.071 247 3	5.828 476 7
199	39601	7 880 599	14.106 736	5.838 272 5
200	40000	8 000 000	14.142 135 6	5.848 035 5
201	40401	8 120 601	14.177 446 9	5.857 766
202	40804	8 242 408	14.212 670 4	5.867 464 3
203	41209	8 365 427	14.247 806 8	5.877 130 7
204	41616	8 489 664	14.282 856 9	5.886 765 3
205	42025	8 615 125	14.317 821 1	5.896 368 5
206	42436	8 741 816	14.352 700 1	5.905 940 6
207	42849	8 869 743	14.387 494 6	5.915 481 7
208	43264	8 998 912	14.422 205 1	5.924 992 1
209	43681	9 129 329	14.456 832 3	5.934 472 1
210	44100	9 261 000	14.491 376 7	5.943 922
211	44521	9 393 931	14.525 839	5.953 341 8
212	44944	9 528 128	14.560 219 8	5.962 732
213	45369	9 663 597	14.594 519 5	5.972 092 6
214	45796	9 800 344	14.628 738 8	5.981 424
215	46225	9 938 375	14.662 878 3	5.990 726 4
216	46656	10 077 696	14.696 938 5	6
217	47089	10 218 313	14.730 919 9	6.009 245
218	47524	10 360 232	14.764 823 1	6.018 461 7
219	47961	10 503 459	14.798 648 6	6.027 650 2
220	48400	0 648 000	14.832 397	6.036 810 7
221	48841	0 793 861	14.866 068 7	6.045 943 5
222	49284	10 941 048	14.899 664 4	6.055 048 9

NUMBER.	SQUARE.	CUBE.	SQUARE ROOT.	CUBE ROOT.
223	49 729	11 089 567	14.933 184 5	6.064 127
224	50 176	11 239 424	14.966 629 5	6.073 177 9
225	50 625	11 390 625	15	6.082 202
226	51 076	11 543 176	15.033 296 4	6.091 199 4
227	51 529	11 697 083	15.066 519 2	6.100 170 2
228	51 984	11 852 352	15.099 668 9	6.109 114 7
229	52 441	12 008 989	15.132 746	6.118 033 2
230	52 900	12 167 000	15.165 750 9	6.126 925 7
231	53 361	12 326 391	15.198 684 2	6.135 792 4
232	53 824	12 487 168	15.231 546 2	6.144 633 7
233	54 289	12 649 337	15.264 337 5	6.153 445 5
234	54 756	12 812 904	15.297 058 5	6.162 240 1
235	55 225	12 977 875	15.329 709 7	6.171 005 8
236	55 696	13 144 256	15.362 291 5	6.179 746 6
237	56 169	13 312 053	15.394 804 3	6.188 462 8
238	56 644	13 481 272	15.427 248 6	6.197 154 4
239	57 121	13 651 919	15.459 624 8	6.205 821 8
240	57 600	13 824 000	15.491 933 4	6.214 465
241	58 081	13 997 521	15.524 174 7	6.223 084 3
242	58 564	14 172 488	15.556 349 2	6.231 679 7
243	59 049	14 348 907	15.588 457 3	6.240 251 5
244	59 536	14 526 784	15.620 499 4	6.248 799 8
245	60 025	14 706 125	15.652 475 8	6.257 324 8
246	60 516	14 886 936	15.684 387 1	6.265 826 6
247	61 009	15 069 223	15.716 233 6	6.274 305 4
248	61 504	15 252 992	15.748 015 7	6.282 761 3
249	62 001	15 438 249	15.779 733 8	6.291 194 6
250	62 500	15 625 000	15.811 388 3	6.299 605 3
251	63 001	15 813 251	15.842 979 5	6.307 993 5
252	63 504	16 003 008	15.874 507 9	6.316 359 6
253	64 009	16 194 277	15 905 973 7	6.324 703 5
254	64 516	16 387 064	15.937 377 5	6.333 025 6
255	65 025	16 581 375	15.968 719 4	6.341 325 7
256	65 536	16 777 216	16	6.349 604 2
257	66 049	16 974 593	16.031 21, 5	6.357 861 1
258	66 564	17 173 512	16.062 378 4	6.366 096 8
259	67 081	17 373 979	16.093 476 9	6.374 311 1
260	67 600	17 576 000	16.124 515 5	6.382 504 3
261	68 121	17 779 581	16.155 494 4	6.390 676 5
262	68 644	17 984 728	16.186 414 1	6.398 827 9
263	69 169	18 191 447	16.217 274 7	6.406 958 5
264	69 696	18 399 744	16.248 076 8	6.415 068 7
265	70 225	18 609 625	16.278 820 6	6.423 158 3
266	70 756	18 821 096	16.309 506 4	6.431 227 6
267	71 289	19 034 163	16.340 134 6	6.439 276 7
268	71 824	19 248 832	16.370 705 5	6.447 305 7
269	72 361	19 465 109	16.401 219 5	6.455 314 8
270	72 900	19 683 000	16.431 676 7	6.463 304 1
271	73 441	19 902 511	16.462 077 6	6.471 273 6
272	73 984	20 123 648	16.492 422 5	6.479 223 6
273	74 529	20 346 417	16.522 711 6	6.487 154 1
274	75 076	20 570 824	16.552 945 4	6.495 065 3
275	75 625	20 796 875	16.583 124	6.502 957 2
276	76 176	21 024 576	16.613 247 7	6.510 83
277	76 729	21 253 933	16.643 317	6.518 683 9
278	77 284	21 484 952	16.678 333	6.526 518 9

Logarithms of Numbers.

From 1 *to* 10 000.

No.	Logarithm.	No.	Logarithm.	No.	Logarithm.	No.	Logarithm.
1	.0	26	1.414 973	51	1.707 57	76	1.880 814
2	.301 03	27	1.431 364	52	1.716 003	77	1.886 491
3	.477 121	28	1.447 158	53	1.724 276	78	1.892 095
4	.602 06	29	1.462 398	54	1.732 394	79	1.897 627
5	.698 97	30	1.477 121	55	1.740 363	80	1.903 09
6	.778 151	31	1.491 362	56	1.748 188	81	1.908 485
7	.845 098	32	1.505 15	57	1.755 875	82	1.913 814
8	.903 09	33	1.518 514	58	1.763 428	83	1.919 078
9	.954 243	34	1.531 479	59	1.770 852	84	1.924 279
10	1	35	1.544 068	60	1.778 151	85	1.929 419
11	1.041 393	36	1.556 303	61	1.785 33	86	1.934 498
12	1.079 181	37	1.568 202	62	1.792 392	87	1.939 519
13	1.113 943	38	1.579 784	63	1.799 341	88	1.944 483
14	1.146 128	39	1.591 065	64	1.806 18	89	1.949 39
15	1.176 091	40	1.602 06	65	1.812 913	90	1.954 243
16	1.204 12	41	1.612 784	66	1.819 544	91	1.959 041
17	1.230 449	42	1.623 249	67	1.826 075	92	1.963 788
18	1.255 273	43	1.633 468	68	1.832 509	93	1.968 483
19	1.278 754	44	1.643 453	69	1.838 849	94	1.973 128
20	1.301 03	45	1.653 213	70	1.845 098	95	1.977 724
21	1.322 219	46	1.662 758	71	1.851 258	96	1.982 271
22	1.342 423	47	1.672 098	72	1.857 332	97	1.986 772
23	1.361 728	48	1.681 241	73	1.863 323	98	1.991 226
24	1.380 211	49	1.690 196	74	1.869 232	99	1.995 635
25	1.397 94	50	1.698 97	75	1.875 061	100	2

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100	00-	0000	0434	0868	1301	1734	2166	2598	3029	3461	3891	432
101	00-	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	00-	86	9026	9451	9876	—	—	—	—	—	—	425
102	01-	—	—	—	—	03	0724	1147	157	1993	2415	424
103	01-	2837	3259	368	41	4521	494	536	5779	6197	6616	420
104	01-	7033	7451	7868	8284	87	9116	9532	9947	—	—	417
104	02-	—	—	—	—	—	—	—	—	0361	0775	416
105	02-	1189	1603	2016	2428	2841	3252	3664	4075	4486	4896	412
106	02-	5306	5715	6125	6533	6942	735	7757	8164	8571	8978	408
107	02-	9384	9789	—	—	—	—	—	—	—	—	405
107	03-	—	—	0195	06	1004	1408	1812	2216	2619	3021	404
108	03-	3424	3826	4227	4628	5029	543	583	623	6629	7028	400
109	03-	7426	7825	8223	862	9017	9414	9811	—	—	—	398
109	04-	—	—	—	—	—	—	—	0207	0602	0998	397
110	04-	1393	1787	2182	2576	2969	3362	3755	4148	454	4932	393
111	04-	5323	5714	6105	6495	6885	7275	7664	8053	8442	883	389
112	04-	9218	9606	9993	—	—	—	—	—	—	—	388
112	05-	—	—	—	038	0766	1153	1538	1924	2309	2694	386
113	05-	3078	3463	3846	423	4613	4996	5378	576	6142	6524	383
114	05-	6905	7286	7666	8046	8426	8805	9185	9563	9942	—	383
114	06-	—	—	—	—	—	—	—	—	—	032	379
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115	06-	0698	1075	1452	1829	2206	2582	2958	3333	3709	4083	376
116	06-	4458	4832	5206	558	5953	6326	6699	7071	7443	7815	373
117	06-	8186	8557	8928	9298	9668	—	—	—	—	—	380
117	07-	—	—	—	—	—	0038	0407	0776	1145	1514	370
118	07-	1882	225	2617	2985	3352	3718	4085	4451	4816	5182	366
119	07-	5547	5912	6276	664	7004	7368	7731	8094	8457	8819	363
120	07-	9181	9543	9904	—	—	—	—	—	—	—	362
120	08-	—	—	—	0266	0626	0987	1347	1707	2067	2426	360
121	08-	2785	3144	3503	3861	4219	4576	4934	5291	5647	6004	357
122	08-	636	6716	7071	7426	7781	8136	849	8845	9198	9552	355
123	08-	9905	—	—	—	—	—	—	—	—	—	355
123	09-	—	0258	0611	0963	1315	1667	2018	237	2721	3071	353
124	09-	3422	3772	4122	4471	482	5169	5518	5866	6215	6562	349
125	09-	691	7257	7604	7951	8298	8644	899	9335	9681	—	348
125	10-	—	—	—	—	—	—	—	—	—	0026	346
126	10-	0371	0715	1059	1403	1747	2091	2434	2777	3119	3462	343
127	10-	3804	4146	4487	4828	5169	551	5851	6191	6531	6871	341
128	10-	721	7549	7888	8227	8565	8903	9241	9579	9916	—	338
128	11-	—	—	—	—	—	—	—	—	—	0253	337
129	11-	059	0926	1263	1599	1934	227	2605	294	3275	3609	335
130	11-	3943	4277	4611	4944	5278	5611	5943	6276	6608	694	333
131	11-	7271	7603	7934	8265	8595	8926	9256	9586	9915	—	331
131	12-	—	—	—	—	—	—	—	—	—	0245	330
132	12-	0574	0903	1231	156	1888	2216	2544	2871	3198	3525	328
133	12-	3852	4178	4504	483	5156	5481	5806	6131	6456	6781	325
134	12-	7105	7429	7753	8076	8399	8722	9045	9368	969	—	323
134	13-	—	—	—	—	—	—	—	—	—	0012	323
135	13-	0334	0655	0977	1298	1619	1939	226	258	29	3219	321
136	13-	3539	3858	4177	4496	4814	5133	5451	5769	6986	6403	318
137	13-	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
138	13-	9879	—	—	—	—	—	—	—	—	—	315
138	14-	—	0194	0508	0822	1136	145	1763	2076	2389	2702	314
139	14-	3015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
140	14-	6128	6438	6748	7058	7367	7676	7985	8294	8603	8911	309
141	14-	9219	9527	9835	—	—	—	—	—	—	—	308
141	15-	—	—	—	0142	0449	0756	1063	137	1676	1982	307
142	15-	2288	2594	29	3205	351	3815	412	4424	4728	5032	305
143	15-	5336	564	5943	6246	6549	6852	7154	7457	7759	8061	303
144	15-	8362	8664	8965	9266	9567	9868	—	—	—	—	302
144	16-	—	—	—	—	—	—	0168	0469	0769	1068	301
145	16-	1368	1667	1967	2266	2564	2863	3161	346	3758	4055	299
146	16-	4353	465	4947	5244	5541	5838	6134	643	6726	7022	297
147	16-	7317	7613	7908	8203	8497	8792	9086	938	9674	9968	295
148	17-	0262	0555	0848	1141	1434	1726	2019	2311	2603	2895	293
149	17-	3186	3478	3769	406	4351	4641	4932	5222	5512	5802	291
150	17-	6091	6381	667	6959	7248	7536	7825	8113	8401	8689	289
151	17-	8977	9264	9552	9839	—	—	—	—	—	—	287
151	18-	—	—	—	—	0126	0413	0699	0986	1272	1558	287
152	18-	1844	2129	2415	27	2985	327	3555	3839	4123	4407	285
153	18-	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
154	18-	7521	7803	8084	8366	8647	8928	9209	949	9771	—	281
154	19-	—	—	—	—	—	—	—	—	—	0051	281
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155	19-	0332	0612	0892	1171	1451	173	201	2289	2567	2846	279
156	19-	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
157	19-	59	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
158	19-	8657	8932	9206	9481	9755	—	—	—	—	—	275
158	20-	—	—	—	—	—	0029	0303	0577	085	1124	274
159	20-	1397	167	1943	2216	2488	2761	3033	3305	3577	3848	272
160	20	412	4391	4663	4934	5204	5475	5746	6016	6286	6556	271
161	20·	6826	7096	7365	7634	7904	8173	8441	871	8979	9247	269
162	20	9515	9783	—	—	—	—	—	—	—	—	268
162	21-	—	—	0051	0319	0586	0853	1121	1388	1654	1921	267
163	21-	2188	2454	272	2986	3252	3518	3783	4049	4314	4579	266
164	21-	4844	5109	5373	5638	5902	6166	643	6694	6957	7221	264
165	21-	7484	7747	801	8273	8536	8798	906	9323	9585	9846	262
166	22·	0108	037	0631	0892	1153	1414	1675	1936	2196	2456	261
167	22-	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
168	22	5309	5568	5826	6084	6342	66	6858	7115	7372	763	258
169	22-	7887	8144	84	8657	8913	917	9426	9682	9938	—	257
169	23-	—	—	—	—	—	—	—	—	—	0193	256
170	23	0449	0704	096	1215	147	1724	1979	2234	2488	2742	255
171	23-	2996	325	3504	3757	4011	4264	4517	477	5023	5276	253
172	23-	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
173	23-	8046	8297	8548	8799	9049	9299	955	98	—	—	251
173	24-	—	—	—	—	—	—	—	—	005	03	250
174	24·	0549	0799	1048	1297	1546	1795	2044	2293	2541	279	249
175	24-	3038	3286	3534	3782	403	4277	4525	4772	5019	5266	248
176	24-	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
177	24-	7973	8219	8464	8709	8954	9198	9443	9687	9932	—	246
177	25-	—	—	—	—	—	—	—	—	—	0176	245
178	25-	042	0664	0908	1151	1395	1638	1881	2125	2368	261	243
179	25-	2853	3096	3338	358	3822	4064	4306	4548	479	5031	242
180	25-	5273	5514	5755	5996	6237	6477	6718	6958	7198	7439	241
181	25-	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	26-	0071	031	0548	0787	1025	1263	1501	1739	1976	2214	238
183	26-	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
184	26-	4818	5054	529	5525	5761	5996	6232	6467	6702	6937	235
185	26-	7172	7406	7641	7875	811	8344	8578	8812	9046	9279	234
186	26-	9513	9746	998	—	—	—	—	—	—	—	234
186	27-	—	—	—	0213	0446	0679	0912	1144	1377	1609	233
187	27-	1842	2074	2306	2538	277	3001	3233	3464	3696	3927	232
188	27-	4158	4389	462	485	5081	5311	5542	5772	6002	6232	230
189	27-	6462	6692	6921	7151	738	7609	7838	8067	8296	8525	229
190	27-	8754	8982	9211	9439	9667	9895	—	—	—	—	228
190	28-	—	—	—	—	—	—	0123	0351	0578	0806	228
191	28-	1033	1261	1488	1715	1942	2169	2396	2622	2849	3075	227
192	28-	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	28-	5557	5782	6007	6232	6456	6681	6905	713	7354	7578	225
194	28-	7802	8026	8249	8473	8696	892	9143	9366	9589	9812	223
195	29-	0035	0257	048	0702	0925	1147	1369	1591	1813	2034	222
196	29-	2256	2478	2699	292	3141	3363	3584	3804	4025	4246	221
197	29-	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
198	29-	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
199	29-	8853	9071	9289	9507	9725	9943	—	—	—	—	218
199	30-	—	—	—	—	—	—	0161	0378	0595	0813	218
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200	30-	103	1247	1464	1681	1898	2114	2331	2547	2764	298	217
201	30-	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
202	30-	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
203	30-	7496	771	7924	8137	8351	8564	8778	8991	9204	9417	213
204	30-	963	9843	—	—	—	—	—	—	—	—	213
204	31-	—	—	0056	0268	0481	0693	0906	1118	133	1542	212
205	31-	1754	1966	2177	2389	26	2812	3023	3234	3445	3656	211
206	31-	3867	4078	4289	4499	471	492	513	534	5551	576	210
207	31-	597	618	639	6599	6809	7018	7227	7436	7646	7854	209
208	31-	8063	8272	8481	8689	8898	9106	9314	9522	973	9938	208
209	32-	0146	0354	0562	0769	0977	1184	1391	1598	1805	2012	207
210	32-	2219	2426	2633	2839	3046	3252	3458	3665	3871	4077	206
211	32-	4282	4488	4694	4899	5105	531	5516	5721	5926	6131	205
212	32-	6336	6541	6745	695	7155	7359	7563	7767	7972	8176	204
213	32-	838	8583	8787	8991	9194	9398	9601	9805	—	—	204
213	33-	—	—	—	—	—	—	—	—	0008	0211	203
214	33-	0414	0617	0819	1022	1225	1427	163	1832	2034	2236	202
215	33-	2438	264	2842	3044	3246	3447	3649	385	4051	4253	202
216	33-	4454	4655	4856	5057	5257	5458	5658	5859	6059	626	201
217	33-	646	666	686	706	726	7459	7659	7858	8058	8257	200
218	33-	8456	8656	8855	9054	9253	9451	965	9849	—	—	200
218	34-	—	—	—	—	—	—	—	—	0047	0246	199
219	34-	0444	0642	0841	1039	1237	1435	1632	183	2028	2225	198
220	34-	2423	262	2817	3014	3212	3409	3606	3802	3999	4196	197
221	34-	4392	4589	4785	4981	5178	5374	557	5766	5962	6157	196
222	34-	6353	6549	6744	6939	7135	733	7525	772	7915	811	195
223	34-	8305	85	8694	8889	9083	9278	9472	9666	986	—	194
223	35-	—	—	—	—	—	—	—	—	—	0054	194
224	35-	0248	0442	0636	0829	1023	1216	141	1603	1796	1989	193
225	35-	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
226	35-	4108	4301	4493	4685	4876	5068	526	5452	5643	5834	192
227	35-	6026	6217	6408	6599	679	6981	7172	7363	7554	7744	191
228	35-	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
229	35-	9835	—	—	—	—	—	—	—	—	—	189
229	36-	—	0025	0215	0404	0593	0783	0972	1161	135	1539	189
230	36-	1728	1917	2105	2294	2482	2671	2859	3048	3236	3424	188
231	36-	3612	38	3988	4176	4363	4551	4739	4926	5113	5301	188
232	36-	5488	5675	5862	6049	6236	6423	661	6796	6983	7169	187
233	36-	7356	7542	7729	7915	8101	8287	8473	8659	8845	903	186
234	36-	9216	9401	9587	9772	9958	—	—	—	—	—	186
234	37-	—	—	—	—	—	0143	0328	0513	0698	0883	185
235	37-	1068	1253	1437	1622	1806	1991	2175	236	2544	2728	184
236	37-	2912	3096	328	3464	3647	3831	4015	4198	4382	4565	184
237	37-	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	37-	6577	6759	6942	7124	7306	7488	767	7852	8034	8216	182
239	37-	8398	858	8761	8943	9124	9306	9487	9668	9849	—	182
239	38-	—	—	—	—	—	—	—	—	—	003	181
240	38-	0211	0392	0573	0754	0934	1115	1296	1476	1656	1837	181
241	38-	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
242	38-	3815	3995	4174	4353	4533	4712	4891	507	5249	5428	179
243	38-	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
244	38-	739	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
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245	38-	9166	9343	952	9698	9875	—	—	—	—	—	177
245	39-	—	—	—	—	—	0051	0228	0405	0582	0759	177
246	39-	0935	1112	1288	1464	1641	1817	1993	2169	2345	2521	176
247	39-	2697	2873	3048	3224	34	3575	3751	3926	4101	4277	176
248	39-	4452	4627	4802	4977	5152	5326	5501	5676	585	6025	175
249	39-	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
250	39-	794	8114	8287	8461	8634	8808	8981	9154	9328	9501	173
251	39-	9674	9847	—	—	—	—	—	—	—	—	173
251	40-	—	—	002	0192	0365	0538	0711	0883	1056	1228	173
252	40-	1401	1573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	40-	3121	3292	3464	3635	3807	3978	4149	432	4492	4663	171
254	40-	4834	5005	5176	5346	5517	5688	5858	6029	6199	637	171
255	40-	654	671	6881	7051	7221	7391	7561	7731	7901	807	170
256	40-	824	841	8579	8749	8918	9087	9257	9426	9595	9764	169
257	40-	9933	—	—	—	—	—	—	—	—	—	169
257	41-	—	0102	0271	044	0609	0777	0946	1114	1283	1451	169
258	41-	162	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
259	41-	33	3467	3635	3803	397	4137	4305	4472	4639	4806	167
260	41-	4973	514	5307	5474	5641	5808	5974	6141	6308	6474	167
261	41-	6641	6807	6973	7139	7306	7472	7638	7804	797	8135	166
262	41-	8301	8467	8633	8798	8964	9129	9295	946	9625	9791	165
263	41-	9956	—	—	—	—	—	—	—	—	—	165
263	42-	—	0121	0286	0451	0616	0781	0945	111	1275	1439	165
264	42-	1604	1768	1933	2097	2261	2426	259	2754	2918	3082	164
265	42-	3246	341	3574	3737	3901	4065	4228	4392	4555	4718	164
266	42-	4882	5045	5208	5371	5534	5697	586	6023	6186	6349	163
267	42-	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
268	42-	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
269	42-	9752	9914	—	—	—	—	—	—	—	—	162
269	43-	—	—	0075	0236	0398	0559	072	0881	1042	1203	161
270	43-	1364	1525	1685	1846	2007	2167	2328	2488	2649	2809	161
271	43-	2969	313	329	345	361	377	393	409	4249	4409	160
272	43-	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
273	43-	6163	6322	6481	664	6799	6957	7116	7275	7433	7592	159
274	43-	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
275	43-	9333	9491	9648	9806	9964	—	—	—	—	—	158
275	44-	—	—	—	—	—	0122	0279	0437	0594	0752	158
276	44-	0909	1066	1224	1381	1538	1695	1852	2009	2166	2323	157
277	44-	248	2637	2793	295	3106	3263	3419	3576	3732	3889	157
278	44-	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
279	44-	5604	576	5915	6071	6226	6382	6537	6692	6848	7003	155
280	44-	7158	7313	7468	7623	7778	7933	8088	8242	8397	8552	155
281	44-	8706	8861	9015	917	9324	9478	9633	9787	9941	—	154
281	45-	—	—	—	—	—	—	—	—	—	0095	154
282	45-	0249	0403	0557	0711	0865	1018	1172	1326	1479	1633	154
283	45-	1786	194	2093	2247	24	2553	2706	2859	3012	3165	153
284	45-	3318	3471	3624	3777	393	4082	4235	4387	454	4692	153
285	45-	4845	4997	515	5302	5454	5606	5758	591	6062	6214	152
286	45-	6366	6518	667	6821	6973	7125	7276	7428	7579	7731	152
287	45-	7882	8033	8184	8336	8487	8638	8789	894	9091	9242	151
288	45-	9392	9543	9694	9845	9995	—	—	—	—	—	151
288	46-	—	—	—	—	—	0146	0296	0447	0597	0748	151
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290	46-	2398	2548	2697	2847	2997	3146	3296	3445	3594	3744	150
291	46-	3893	4042	4191	434	449	4639	4788	4936	5085	5234	149
292	46-	5383	5532	568	5829	5977	6126	6274	6423	6571	6719	149
293	46-	6868	7016	7164	7312	746	7608	7756	7904	8052	82	148
294	46-	8347	8495	8643	879	8938	9085	9233	938	9527	9675	148
295	46-	9822	9969	—	—	—	—	—	—	—	—	147
295	47-	—	—	0116	0263	041	0557	0704	0851	0998	1145	147
296	47-	1292	1438	1585	1732	1878	2025	2171	2318	2464	261	146
297	47-	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
298	47-	4216	4362	4508	4653	4799	4944	509	5235	5381	5526	146
299	47-	5671	5816	5962	6107	6252	6397	6542	6687	6832	6976	145
300	47-	7121	7266	7411	7555	77	7844	7989	8133	8278	8422	145
301	47-	8566	8711	8855	8999	9143	9287	9431	9575	9719	9863	144
302	48-	0007	0151	0294	0438	0582	0725	0869	1012	1156	1299	144
303	48-	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
304	48-	2874	3016	3159	3302	3445	3587	373	3872	4015	4157	143
305	48-	43	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
306	48-	5721	5863	6005	6147	6289	643	6572	6714	6855	6997	142
307	48-	7138	728	7421	7563	7704	7845	7986	8127	8269	841	141
308	48-	8551	8692	8833	8974	9114	9255	9396	9537	9677	9818	141
309	48-	9958	—	—	—	—	—	—	—	—	—	140
309	49-	—	0099	0239	038	052	0661	0801	0941	1081	1222	140
310	49-	1362	1502	1642	1782	1922	2062	2201	2341	2481	2621	140
311	49-	276	29	304	3179	3319	3458	3597	3737	3876	4015	139
312	49-	4155	4294	4433	4572	4711	485	4989	5128	5267	5406	139
313	49-	5544	5683	5822	596	6099	6238	6376	6515	6653	6791	139
314	49-	693	7068	7206	7344	7483	7621	7759	7897	8035	8173	138
315	49-	8311	8448	8586	8724	8862	8999	9137	9275	9412	955	138
316	49-	9687	9824	9962	—	—	—	—	—	—	—	137
316	50-	—	—	—	0099	0236	0374	0511	0648	0785	0922	137
317	50-	1059	1196	1333	147	1607	1744	188	2017	2154	2291	137
318	50-	2427	2564	27	2837	2973	3109	3246	3382	3518	3655	136
319	50-	3791	3927	4063	4199	4335	4471	4607	4743	4878	5014	136
320	50-	515	5286	5421	5557	5693	5828	5964	6099	6234	637	136
321	50-	6505	664	6776	6911	7046	7181	7316	7451	7586	7721	135
322	50-	7856	7991	8126	826	8395	853	8664	8799	8934	9068	135
323	50-	9203	9337	9471	9606	974	9874	—	—	—	—	134
323	51-	—	—	—	—	—	—	0009	0143	0277	0411	134
324	51-	0545	0679	0813	0947	1081	1215	1349	1482	1616	175	134
325	51-	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
326	51-	3218	3351	3484	3617	375	3883	4016	4149	4282	4415	133
327	51-	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
328	51-	5874	6006	6139	6271	6403	6535	6668	68	6932	7064	132
329	51-	7196	7328	746	7592	7724	7855	7987	8119	8251	8382	132
330	51-	8514	8646	8777	8909	904	9171	9303	9434	9566	9697	131
331	51-	9828	9959	—	—	—	—	—	—	—	—	131
331	52-	—	—	009	0221	0353	0484	0615	0745	0876	1007	131
332	52-	1138	1269	14	153	1661	1792	1922	2053	2183	2314	131
333	52-	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	52-	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
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335	52-	5045	5174	5304	5434	5563	5693	5822	5951	6081	621	129
336	52-	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	52-	763	7759	7888	8016	8145	8274	8402	8531	866	8788	129
338	52-	8917	9045	9174	9302	943	9559	9687	9815	9943	—	128
338	53-	—	—	—	—	—	—	—	—	—	0072	128
339	53-	02	0328	0456	0584	0712	084	0968	1096	1223	1351	128
340	53-	1479	1607	1734	1862	199	2117	2245	2372	25	2627	128
341	53-	2754	2882	3009	3136	3264	3391	3518	3645	3772	3899	127
342	53-	4026	4153	428	4407	4534	4661	4787	4914	5041	5167	127
343	53-	5294	5421	5547	5674	58	5927	6053	618	6306	6432	126
344	53-	6558	6685	6811	6937	7063	7189	7315	7441	7567	7693	126
345	53-	7819	7945	8071	8197	8322	8448	8574	8699	8825	8951	126
346	53-	9076	9202	9327	9452	9578	9703	9829	9954	—	—	126
346	54-	—	—	—	—	—	—	—	—	0079	0204	125
347	54-	0329	0455	058	0705	083	0955	108	1205	133	1454	125
348	54-	1579	1704	1829	1953	2078	2203	2327	2452	2576	2701	125
349	54-	2825	295	3074	3199	3323	3447	3571	3696	382	3944	124
350	54-	4068	4192	4316	444	4564	4688	4812	4936	506	5183	124
351	54-	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
352	54-	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
353	54-	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
354	54-	9003	9126	9249	9371	9494	9616	9739	9861	9984	—	123
354	55-	—	—	—	—	—	—	—	—	—	0106	123
355	55-	0228	0351	0473	0595	0717	084	0962	1084	1206	1328	122
356	55-	145	1572	1694	1816	1938	206	2181	2303	2425	2547	122
357	55-	2668	279	2911	3033	3155	3276	3398	3519	364	3762	121
358	55-	3883	4004	4126	4247	4368	4489	461	4731	4852	4973	121
359	55-	5094	5215	5336	5457	5578	5699	582	594	6061	6182	121
360	55-	6303	6423	6544	6664	6785	6905	7026	7146	7267	7387	120
361	55-	7507	7627	7748	7868	7988	8108	8228	8349	8469	8589	120
362	55-	8709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	55-	9907	—	—	—	—	—	—	—	—	—	120
363	56-	—	0026	0146	0265	0385	0504	0624	0743	0863	0982	119
364	56-	1101	1221	134	1459	1578	1698	1817	1936	2055	2174	119
365	56-	2293	2412	2531	265	2769	2887	3006	3125	3244	3362	119
366	56-	3481	36	3718	3837	3955	4074	4192	4311	4429	4548	119
367	56-	4666	4784	4903	5021	5139	5257	5376	5494	5612	573	118
368	56-	5848	5966	6084	.6202	632	6437	6555	6673	6791	6909	118
369	56-	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	56-	8202	8319	8436	8554	8671	8788	8905	9023	914	9257	117
371	56-	9374	9491	9608	9725	9842	9959	—	—	—	—	117
371	57-	—	—	—	—	—	—	0076	0193	0309	0426	117
372	57-	0543	066	0776	0893	101	1126	1243	1359	1476	1592	117
373	57-	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	57-	2872	2988	3104	322	3336	3452	3568	3684	38	3915	116
375	57-	4031	4147	4263	4379	4494	461	4726	4841	4957	5072	116
376	57-	5188	5303	5419	5534	565	5765	588	5996	6111	6226	115
377	57-	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
378	57-	7492	7607	7722	7836	7951	8066	8181	8295	841	8525	115
379	57-	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
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380	57-	9784	9898	—	—	—	—	—	—	—	—	114
380	58-	—	—	0012	0126	0241	0355	0469	0583	0697	0811	114
381	58-	0925	1039	1153	1267	1381	1495	1608	1722	1836	195	114
382	58-	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
383	58-	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	58-	4331	4444	4557	467	4783	4896	5009	5122	5235	5348	113
385	58-	5461	5574	5686	5799	5912	6024	6137	625	6362	6475	113
386	58-	6587	67	6812	6925	7037	7149	7262	7374	7486	7599	112
387	58-	7711	7823	7935	8047	816	8272	8384	8496	8608	872	112
388	58-	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
389	58-	995	—	—	—	—	—	—	—	—	—	112
389	59-	—	0061	0173	0284	0396	0507	0619	073	0842	0953	112
390	59-	1065	1176	1287	1399	151	1621	1732	1843	1955	2066	111
391	59-	2177	2288	2399	251	2621	2732	2843	2954	3064	3175	111
392	59-	3286	3397	3508	3618	3729	384	395	4061	4171	4282	111
393	59-	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	59-	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
395	59-	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	110
396	59-	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	110
397	59-	8791	89	9009	9119	9228	9337	9446	9556	9665	9774	109
398	59-	9883	9992	—	—	—	—	—	—	—	—	109
398	60-	—	—	0101	021	0319	0428	0537	0646	0755	0864	109
399	60-	0973	1082	1191	1299	1408	1517	1625	1734	1843	1951	109
400	60-	206	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
401	60-	3144	3253	3361	3469	3577	3686	3794	3902	401	4118	108
402	60-	4226	4334	4442	455	4658	4766	4874	4982	5089	5197	108
403	60-	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
404	60-	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	60-	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
406	60-	8526	8633	874	8847	8954	9061	9167	9274	9381	9488	107
407	60-	9594	9701	9808	9914	—	—	—	—	—	—	107
407	61-	—	—	—	—	0021	0128	0234	0341	0447	0554	107
408	61-	066	0767	0873	0979	1086	1192	1298	1405	1511	1617	106
409	61-	1723	1829	1936	2042	2148	2254	236	2466	2572	2678	106
410	61-	2784	289	2996	3102	3207	3313	3419	3525	363	3736	106
411	61-	3842	3947	4053	4159	4264	437	4475	4581	4686	4792	106
412	61-	4897	5003	5108	5213	5319	5424	5529	5634	574	5845	105
413	61-	595	6055	616	6265	637	6476	6581	6686	679	6895	105
414	61-	7	7105	721	7315	742	7525	7629	7734	7839	7943	105
415	61-	8048	8153	8257	8362	8466	8571	8676	878	8884	8989	105
416	61-	9093	9198	9302	9406	9511	9615	9719	9824	9928	—	105
416	62-	—	—	—	—	—	—	—	—	—	9032	104
417	62-	0136	024	0344	0448	0552	0656	076	0864	0968	1072	104
418	62-	1176	128	1384	1488	1592	1695	1799	1903	2007	211	104
419	62-	2214	2318	2421	2525	2628	2732	2835	2939	3042	3146	104
420	62-	3249	3353	3456	3559	3663	3766	3869	3973	4076	4179	103
421	62-	4282	4385	4488	4591	4695	4798	4901	5004	5107	521	103
422	62-	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
423	62-	634	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
424	62-	7366	7468	7571	7673	7775	7878	798	8082	8185	8287	102
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425	62-	8389	8491	8593	8695	8797	89	9002	9104	9206	9308	102
426	62-	941	9512	9613	9715	9817	9919	—	—	—	—	102
426	63-	—	—	—	—	—	—	0021	0123	0224	0326	102
427	63-	0428	053	0631	0733	0835	0936	1038	1139	1241	1342	102
428	63-	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	63-	2457	2559	266	2761	2862	2963	3064	3165	3266	3367	101
430	63-	3468	3569	367	3771	3872	3973	4074	4175	4276	4376	101
431	63-	4477	4578	4679	4779	488	4981	5081	5182	5283	5383	101
432	63-	5484	5584	5685	5785	5886	5986	6087	6187	6287	6388	100
433	63-	6488	6588	6688	6789	6889	6989	7089	7189	729	739	100
434	63-	749	759	769	779	789	799	809	819	829	8389	100
435	63-	8489	8589	8689	8789	8888	8988	9088	9188	9287	9387	100
436	63-	9486	9586	9686	9785	9885	9984	—	—	—	—	100
436	64-	—	—	—	—	—	—	0084	0183	0283	0382	99
437	64-	0481	0581	068	0779	0879	0978	1077	1177	1276	1375	99
438	64-	1474	1573	1672	1771	1871	197	2069	2168	2267	2366	99
439	64-	2465	2563	2662	2761	286	2959	3058	3156	3255	3354	99
440	64-	3453	3551	365	3749	3847	3946	4044	4143	4242	434	99
441	64-	4439	4537	4636	4734	4832	4931	5029	5127	5226	5324	98
442	64-	5422	5521	5619	5717	5815	5913	6011	611	6208	6306	98
443	64-	6404	6502	66	6698	6796	6894	6992	7089	7187	7285	98
444	64-	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
445	64-	836	8458	8555	8653	875	8848	8945	9043	914	9237	97
446	64-	9335	9432	953	9627	9724	9821	9919	—	—	—	97
446	65-	—	—	—	—	—	—	—	0016	0113	021	97
447	65-	0308	0405	0502	0599	0696	0793	089	0987	1084	1181	97
448	65-	1278	1375	1472	1569	1666	1762	1859	1956	2053	215	97
449	65-	2246	2343	244	2536	2633	273	2826	2923	3019	3116	97
450	65-	3213	3309	3405	3502	3598	3695	3791	3888	3984	408	96
451	65-	4177	4273	4369	4465	4562	4658	4754	485	4946	5042	96
452	65-	5138	5235	5331	5427	5523	5619	5715	581	5906	6002	96
453	65-	6098	6194	629	6386	6482	6577	6673	6769	6864	696	96
454	65-	7056	7152	7247	7343	7438	7534	7629	7725	782	7916	96
455	65-	8011	8107	8202	8298	8393	8488	8584	8679	8774	887	95
456	65-	8965	906	9155	925	9346	9441	9536	9631	9726	9821	95
457	65-	9916	—	—	—	—	—	—	—	—	—	95
457	66-	—	0011	0106	0201	0296	0391	0486	0581	0676	0771	95
458	66-	0865	096	1055	115	1245	1339	1434	1529	1623	1718	95
459	66-	1813	1907	2002	2096	2191	2286	238	2475	2569	2663	95
460	66-	2758	2852	2947	3041	3135	323	3324	3418	3512	3607	94
461	66-	3701	3795	3889	3983	4078	4172	4266	436	4454	4548	94
462	66-	4642	4736	483	4924	5018	5112	5206	5299	5393	5487	94
463	66-	5581	5675	5769	5862	5956	605	6143	6237	6331	6424	94
464	66-	6518	6612	6705	6799	6892	6986	7079	7173	7266	736	94
465	66-	7453	7546	764	7733	7826	792	8013	8106	8199	8293	93
466	66-	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
467	66-	9317	941	9503	9596	9689	9782	9875	9967	—	—	93
467	67-	—	—	—	—	—	—	—	—	006	0153	93
468	67-	0246	0339	0431	0524	0617	071	0802	0895	0988	108	93
469	67-	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
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470	67-	2098	219	2283	2375	2467	256	2652	2744	2836	2929	92
471	67-	3021	3113	3205	3297	339	3482	3574	3666	3758	385	92
472	67-	3942	4034	4126	4218	431	4402	4494	4586	4677	4769	92
473	67-	4861	4953	5045	5137	5228	532	5412	5503	5595	5687	92
474	67-	5778	587	5962	6053	6145	6236	6328	6419	6511	6602	92
475	67-	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	67-	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	67-	8518	8609	87	8791	8882	8973	9064	9155	9246	9337	91
478	67-	9428	9519	961	97	9791	9882	9973	—	—	—	91
478	68-	—	—	—	—	—	—	—	0063	0154	0245	91
479	68-	0336	0426	0517	0607	0698	0789	0879	097	106	1151	91
480	68-	1241	1332	1422	1513	1603	1693	1784	1874	1964	2055	90
481	68-	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
482	68-	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
483	68-	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
484	68-	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
485	68-	5742	5831	5921	601	61	6189	6279	6368	6458	6547	89
486	68-	6636	6726	6815	6904	6994	7083	7172	7261	7351	744	89
487	68-	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	68-	842	8509	8598	8687	8776	8865	8953	9042	9131	922	89
489	68-	9309	9398	9486	9575	9664	9753	9841	993	—	—	89
489	69-	—	—	—	—	—	—	—	—	0019	0107	89
490	69-	0196	0285	0373	0462	055	0639	0728	0816	0905	0993	89
491	69-	1081	117	1258	1347	1435	1524	1612	17	1789	1877	88
492	69-	1965	2053	2142	223	2318	2406	2494	2583	2671	2759	88
493	69-	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
494	69-	3727	3815	3903	3991	4078	4166	4254	4342	443	4517	88
495	69-	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
496	69-	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
497	69-	6356	6444	6531	6618	6706	6793	688	6968	7055	7142	87
498	69-	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
499	69-	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
500	69-	897	9057	9144	9231	9317	9404	9491	9578	9664	9751	87
501	69-	9838	9924	—	—	—	—	—	—	—	—	87
501	70-	—	—	0011	0098	0184	0271	0358	0444	0531	0617	87
502	70-	0704	079	0877	0963	105	1136	1222	1309	1395	1482	86
503	70-	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	70-	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
505	70-	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
506	70-	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	70-	5008	5094	5179	5265	535	5436	5522	5607	5693	5778	86
508	70-	5864	5949	6035	612	6206	6291	6376	6462	6547	6632	85
509	70-	6718	6803	6888	6974	7059	7144	7229	7315	74	7485	85
510	70-	757	7655	774	7826	7911	7996	8081	8166	8251	8336	85
511	70-	8421	8506	8591	8676	8761	8846	8931	9015	91	9185	85
512	70-	927	9355	944	9524	9609	9694	9779	9863	9948	—	85
512	71-	—	—	—	—	—	—	—	—	—	0033	85
513	71-	0117	0202	0287	0371	0456	054	0625	071	0794	0879	85
514	71-	0963	1048	1132	1217	1301	1385	147	1554	1639	1723	84
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515	71- 1807	1892	1976	206	2144	2229	2313	2397	2481	2566	84
516	71- 265	2734	2818	2902	2986	307	3154	3238	3323	3407	84
517	71- 3491	3575	3659	3742	3826	391	3994	4078	4162	4246	84
518	71- 433	4414	4497	4581	4665	4749	4833	4916	5	5084	84
519	71- 5167	5251	5335	5418	5502	5586	5669	5753	5836	592	84
520	71- 6003	6087	617	6254	6337	6421	6504	6588	6671	6754	83
521	71- 6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
522	71- 7671	7754	7837	792	8003	8086	8169	8253	8336	8419	83
523	71- 8502	8585	8668	8751	8834	8917	9	9083	9165	9248	83
524	71- 9331	9414	9497	958	9663	9745	9828	9911	9994	—	83
524	72- —	—	—	—	—	—	—	—	—	0077	83
525	72- 0159	0242	0325	0407	049	0573	0655	0738	0821	0903	83
526	72- 0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	72- 1811	1893	1975	2058	214	2222	2305	2387	2469	2552	82
528	72- 2634	2716	2798	2881	2963	3045-	3127	3209	3291	3374	82
529	72- 3456	3538	362	3702	3784	3866	3948	403	4112	4194	82
530	72- 4276	4358	444	4522	4604	4685	4767	4849	4931	5013	82
531	72- 5095	5176	5258	534	5422	5503	5585	5667	5748	583	82
532	72- 5912	5993	6075	6156	6238	632	6401	6483	6564	6646	82
533	72- 6727	6809	689	6972	7053	7134	7216	7297	7379	746	81
534	72- 7541	7623	7704	7785	7866	7948	8029	811	8191	8273	81
535	72- 8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	72- 9165	9246	9327	9408	9489	957	9651	9732	9813	9893	81
537	72- 9974	—	—	—	—	—	—	—	—	—	81
537	73- —	0055	0136	0217	0298	0378	0459	054	0621	0702	81
538	73- 0782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
539	73- 1589	1669	175	183	1911	1991	2072	2152	2233	2313	81
540	73- 2394	2474	2555	2635	2715	2796	2876	2956	3037	3117	80
541	73- 3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
542	73- 3999	4079	416	424	432	44	448	456	464	472	80
543	73- 48	488	496	504	512	52	5279	5359	5439	5519	80
544	73- 5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
545	73- 6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
546	73- 7193	7272	7352	7431	7511	759	767	7749	7829	7908	79
547	73- 7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	73- 8781	886	8939	9018	9097	9177	9256	9335	9414	9493	79
549	73- 9572	9651	9731	981	9889	9968	—	—	—	—	79
549	74- —	—	—	—	—	—	0047	0126	0205	0284	79
550	74- 0363	0442	0521	06	0678	0757	0836	0915	0994	1073	79
551	74- 1152	123	1309	1388	1467	1546	1624	1703	1782	186	79
552	74- 1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	74- 2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	74- 351	3588	3667	3745	3823	3902	398	4058	4136	4215	78
555	74- 4293	4371	4449	4528	4606	4684	4762	484	4919	4997	78
556	74- 5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	74- 5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	74- 6634	6712	679	6868	6945	7023	7101	7179	7256	7334	78
559	74- 7412	7489	7567	7645	7722	78	7878	7955	8033	811	78
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560	74-	8188	8266	8343	8421	8498	8576	8653	8731	8808	8885	77
561	74-	8963	904	9118	9195	9272	935	9427	9504	9582	9659	77
562	74-	9736	9814	9891	9968	—	—	—	—	—	—	77
562	75-	—	—	—	—	0045	0123	02	0277	0354	0431	77
563	75-	0508	0586	0663	074	0817	0894	0971	1048	1125	1202	77
564	75-	1279	1356	1433	151	1587	1664	1741	1818	1895	1972	77
565	75-	2048	2125	2202	2279	2356	2433	2509	2586	2663	274	77
566	75-	2816	2893	297	3047	3123	32	3277	3353	343	3506	77
567	75-	3583	366	3736	3813	3889	3966	4042	4119	4195	4272	77
568	75-	4348	4425	4501	4578	4654	473	4807	4883	496	5036	76
569	75-	5112	5189	5265	5341	5417	5494	557	5646	5722	5799	76
570	75-	5875	5951	6027	6103	618	6256	6332	6408	6484	656	76
571	75-	6636	6712	6788	6864	694	7016	7092	7168	7244	732	76
572	75-	7396	7472	7548	7624	77	7775	7851	7927	8003	8079	76
573	75-	8155	823	8306	8382	8458	8533	8609	8685	8761	8836	76
574	75-	8912	8988	9063	9139	9214	929	9366	9441	9517	9592	76
575	75-	9668	9743	9819	9894	997	—	—	—	—	—	76
575	76-	—	—	—	—	—	0045	0121	0196	0272	0347	75
576	76-	0422	0498	0573	0649	0724	0799	0875	095	1025	1101	75
577	76-	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	76-	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
579	76-	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75
580	76-	3428	3503	3578	3653	3727	3802	3877	3952	4027	4101	75
581	76-	4176	4251	4326	44	4475	455	4624	4699	4774	4848	75
582	76-	4923	4998	5072	5147	5221	5296	537	5445	552	5594	75
583	76-	5669	5743	5818	5892	5966	6041	6115	619	6264	6338	74
584	76-	6413	6487	6562	6636	671	6785	6859	6933	7007	7082	74
585	76-	7156	723	7304	7379	7453	7527	7601	7675	7749	7823	74
586	76-	7898	7972	8046	812	8194	8268	8342	8416	849	8564	74
587	76-	8638	8712	8786	886	8934	9008	9082	9156	923	9303	74
588	76-	9377	9451	9525	9599	9673	9746	982	9894	9968	—	74
588	77-	—	—	—	—	—	—	—	—	—	0042	74
589	77-	0115	0189	0263	0336	041	0484	0557	0631	0705	0778	74
590	77-	0852	0926	0999	1073	1146	122	1293	1367	144	1514	74
591	77-	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	77-	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	77-	3055	3128	3201	3274	3348	3421	3494	3567	364	3713	73
594	77-	3786	386	3933	4006	4079	4152	4225	4298	4371	4444	73
595	77-	4517	459	4663	4736	4809	4882	4955	5028	51	5173	73
596	77-	5246	5319	5392	5465	5538	561	5683	5756	5829	5902	73
597	77-	5974	6047	612	6193	6265	6338	6411	6483	6556	6629	73
598	77-	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
599	77-	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
600	77-	8151	8224	8296	8368	8441	8513	8585	8658	873	8802	72
601	77-	8874	8947	9019	9091	9163	9236	9308	938	9452	9524	72
602	77-	9596	9669	9741	9813	9885	9957	—	—	—	—	72
602	78-	—	—	—	—	—	—	0029	0101	0173	0245	72
603	78-	0317	0389	0461	0533	0605	0677	0749	0821	0893	0965	72
604	78-	1037	1109	1181	1253	1324	1396	1468	154	1612	1684	72
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605	78-	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	78-	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	78-	3189	326	3332	3403	3475	3546	3618	3689	3761	3832	71
608	78-	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	78-	4617	4689	476	4831	4902	4974	5045	5116	5187	5259	71
610	78-	533	5401	5472	5543	5615	5686	5757	5828	5899	597	71
611	78-	6041	6112	6183	6254	6325	6396	6467	6538	6609	668	71
612	78-	6751	6822	6893	6964	7035	7106	7177	7248	7319	739	71
613	78-	746	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	78-	8168	8239	831	8381	8451	8522	8593	8663	8734	8804	71
615	78-	8875	8946	9016	9087	9157	9228	9299	9369	944	951	71
616	78-	9581	9651	9722	9792	9863	9933	—	—	—	—	70
616	79-	—	—	—	—	—	—	0004	0074	0144	0215	70
617	79-	0285	0356	0426	0496	0567	0637	0707	0778	0848	0918	70
618	79-	0988	1059	1129	1199	1269	134	141	148	155	162	70
619	79-	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
620	79-	2392	2462	2532	2602	2672	2742	2812	2882	2952	3022	70
621	79-	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
622	79-	379	386	393	4	407	4139	4209	4279	4349	4418	70
623	79-	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	79-	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	79-	588	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	79-	6574	6644	6713	6782	6852	6921	699	706	7129	7198	69
627	79-	7268	7337	7406	7475	7545	7614	7683	7752	7821	789	69
628	79-	796	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
629	79-	8651	872	8789	8858	8927	8996	9065	9134	9203	9272	69
630	79-	9341	9409	9478	9547	9616	9685	9754	9823	9892	9961	69
631	80-	0029	0098	0167	0236	0305	0373	0442	0511	058	0648	69
632	80-	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
633	80-	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	80-	2089	2158	2226	2295	2363	2432	25	2568	2637	2705	69
635	80-	2774	2842	291	2979	3047	3116	3184	3252	3321	3389	68
636	80-	3457	3525	3594	3662	373	3798	3867	3935	4003	4071	68
637	80-	4139	4208	4276	4344	4412	448	4548	4616	4685	4753	68
638	80-	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	80-	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
640	80-	618	6248	6316	6384	6451	6519	6587	6655	6723	679	68
641	80-	6858	6926	6994	7061	7129	7197	7264	7332	74	7467	68
642	80-	7535	7603	767	7738	7806	7873	7941	8008	8076	8143	68
643	80-	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	80-	8886	8953	9021	9088	9156	9223	929	9358	9425	9492	67
645	80-	956	9627	9694	9762	9829	9896	9964	—	—	—	67
645	81-	—	—	—	—	—	—	—	0031	0098	0165	67
646	81-	0233	03	0367	0434	0501	0569	0636	0703	077	0837	67
647	81-	0904	0971	1039	1106	1173	124	1307	1374	1441	1508	67
648	81-	1575	1642	1709	1776	1843	191	1977	2044	2111	2178	67
649	81-	2245	2312	2379	2445	2512	2579	2646	2713	278	2847	67
650	81-	2913	298	3047	3114	3181	3247	3314	3381	3448	3514	67
651	81-	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
652	81-	4248	4314	4381	4447	4514	4581	4647	4714	478	4847	67
653	81-	4913	498	5046	5113	5179	5246	5312	5378	5445	5511	66
654	81-	5578	5644	5711	5777	5843	591	5976	6042	6109	6175	66
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655	81- 6241	6308	6374	644	6506	6573	6639	6705	6771	6838	66
656	81- 6904	697	7036	7102	7169	7235	7301	7367	7433	7499	66
657	81- 7565	7631	7698	7764	783	7896	7962	8028	8094	816	66
658	81- 8226	8292	8358	8424	849	8556	8622	8688	8754	882	66
659	81- 8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
660	81- 9544	961	9676	9741	9807	9873	9939	—	—	—	66
660	82- —	—	—	—	—	—	—	0004	007	0136	66
661	82- 0201	0267	0333	0399	0464	053	0595	0661	0727	0792	66
662	82- 0858	0924	0989	1055	112	1186	1251	1317	1382	1448	66
663	82- 1514	1579	1645	171	1775	1841	1906	1972	2037	2103	65
664	82- 2168	2233	2299	2364	243	2495	256	2626	2691	2756	65
665	82- 2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	82- 3474	3539	3605	367	3735	38	3865	393	3996	4061	65
667	82- 4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	82- 4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	82- 5426	5491	5556	5621	5686	5751	5815	588	5945	601	65
670	82- 6075	614	6204	6269	6334	6399	6464	6528	6593	6658	65
671	82- 6723	6787	6852	6917	6981	7046	7111	7175	724	7305	65
672	82- 7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	82- 8015	808	8144	8209	8273	8338	8402	8467	8531	8595	64
674	82- 866	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	82- 9304	9368	9432	9497	9561	9625	969	9754	9818	9882	64
676	82- 9947	—	—	—	—	—	—	—	—	—	64
676	83- —	0011	0075	0139	0204	0268	0332	0396	046	0525	64
677	83- 0589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	83- 123	1294	1358	1422	1486	155	1614	1678	1742	1806	64
679	83- 187	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
680	83- 2509	2573	2637	27	2764	2828	2892	2956	302	3083	64
681	83- 3147	3211	3275	3338	3402	3466	353	3593	3657	3721	64
682	83- 3784	3848	3912	3975	4039	4103	4166	423	4294	4357	64
683	83- 4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	83- 5056	512	5183	5247	531	5373	5437	55	5564	5627	63
685	83- 5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	83- 6324	6387	6451	6514	6577	6641	6704	6767	683	6894	63
687	83- 6957	702	7083	7146	721	7273	7336	7399	7462	7525	63
688	83- 7588	7652	7715	7778	7841	7904	7967	803	8093	8156	63
689	83- 8219	8282	8345	8408	8471	8534	8597	866	8723	8786	63
690	83- 8849	8912	8975	9038	9101	9164	9227	9289	9352	9415	63
691	83- 9478	9541	9604	9667	9729	9792	9855	9918	9981	—	63
691	84- —	—	—	—	—	—	—	—	—	0043	63
692	84 0106	0169	0232	0294	0357	042	0482	0545	0608	0671	63
693	84- 0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
694	84- 1359	1422	1485	1547	161	1672	1735	1797	186	1922	63
695	84- 1985	2047	211	2172	2235	2297	236	2422	2484	2547	62
696	84- 2609	2672	2734	2796	2859	2921	2983	3046	3108	317	62
697	84- 3233	3295	3357	342	3482	3544	3606	3669	3731	3793	62
698	84- 3855	3918	398	4042	4104	4166	4229	4291	4353	4415	62
699	84- 4477	4539	4601	4664	4726	4788	485	4912	4974	5036	62
700	84- 5098	516	5222	5284	5346	5408	547	5532	5594	5656	62
701	84- 5718	578	5842	5904	5966	6028	609	6151	6213	6275	62
702	84- 6337	6399	6461	6523	6585	6646	6708	677	6832	6894	62
703	84- 6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	84- 7573	7634	7696	7758	7819	7881	7943	8004	8066	8127	62
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705	84- 8189	8251	8312	8374	8435	8497	8559	862	8682	8743	62
706	84- 8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	84- 9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	85- 0033	0095	0156	0217	0279	034	0401	0462	0524	0585	61
709	85- 0646	0707	0769	083	0891	0952	1014	1075	1136	1197	61
710	85- 1258	132	1381	1442	1503	1564	1625	1686	1747	1809	61
711	85- 187	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
712	85- 248	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
713	85- 309	315	3211	3272	3333	3394	3455	3516	3577	3637	61
714	85- 3698	3759	382	3881	3941	4002	4063	4124	4185	4245	61
715	85- 4306	4367	4428	4488	4549	461	467	4731	4792	4852	61
716	85- 4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	85- 5519	558	564	5701	5761	5822	5882	5943	6003	6064	61
718	85- 6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
719	85- 6729	6789	685	691	697	7031	7091	7152	7212	7272	60
720	85- 7332	7393	7453	7513	7574	7634	7694	7755	7815	7875	60
721	85- 7935	7995	8056	8116	8176	8236	8297	8357	8417	8477	60
722	85- 8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
723	85- 9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
724	85- 9739	9799	9859	9918	9978	—	—	—	—	—	60
724	86- —	—	—	—	—	0038	9098	0158	0218	0278	60
725	86- 0338	0398	0458	0518	0578	0637	0697	0757	0817	0877	60
726	86- 0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	86- 1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	86- 2131	2191	2251	231	237	243	2489	2549	2608	2668	60
729	86- 2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	86- 3323	3382	3442	3501	3561	362	368	3739	3799	3858	59
731	86- 3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	86- 4511	457	463	4689	4748	4808	4867	4926	4985	5045	59
733	86- 5104	5163	5222	5282	5341	54	5459	5519	5578	5637	59
734	86- 5696	5755	5814	5874	5933	5992	6051	611	6169	6228	59
735	86- 6287	6346	6405	6465	6524	6583	6642	6701	676	6819	59
736	86- 6878	6937	6996	7055	7114	7173	7232	7291	735	7409	59
737	86- 7467	7526	7585	7644	7703	7762	7821	788	7939	7998	59
738	86- 8056	8115	8174	8233	8292	835	8409	8468	8527	8586	59
739	86- 8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
740	86- 9232	929	9349	9408	9466	9525	9584	9642	9701	976	59
741	86- 9818	9877	9935	9994	—	—	—	—	—	—	59
741	87- —	—	—	—	0053	0111	017	0228	0287	0345	59
742	87- 0404	0462	0521	0579	0638	0696	0755	0813	0872	093	58
743	87- 0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	87- 1573	1631	169	1748	1806	1865	1923	1981	204	2098	58
745	87- 2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	87- 2739	2797	2855	2913	2972	303	3088	3146	3204	3262	58
747	87- 3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	87- 3902	396	4018	4076	4134	4192	425	4308	4366	4424	58
749	67- 4482	454	4598	4656	4714	4772	483	4888	4945	5003	58
750	87- 5061	5119	5177	5235	5293	5351	5409	5466	5524	5582	58
751	87- 564	5698	5756	5813	5871	5929	5987	6045	6102	616	58
752	87- 6218	6276	6333	6391	6449	6507	6564	6622	668	6737	58
753	87- 6795	6853	691	6968	7026	7083	7141	7199	7256	7314	58
754	87- 7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
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755	87- 7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	87- 8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	87- 9096	9153	9211	9268	9325	9383	944	9497	9555	9612	57
758	87- 9669	9726	9784	9841	9898	9956	—	—	—	—	57
758	88- —	—	—	—	—	—	0013	007	0127	0185	57
759	88- 0242	0299	0356	0413	0471	0528	0585	0642	0699	0756	57
760	88- 0814	0871	0928	0985	1042	1099	1156	1213	1271	1328	57
761	88- 1385	1442	1499	1556	1613	167	1727	1784	1841	1898	57
762	88- 1955	2012	2069	2126	2183	224	2297	2354	2411	2468	57
763	88- 2525	2581	2638	2695	2752	2809	2866	2923	298	3037	57
764	88- 3093	315	3207	3264	3321	3377	3434	3491	3548	3605	57
765	88- 3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	88- 4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	88- 4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	88- 5361	5418	5474	5531	5587	5644	57	5757	5813	587	57
769	88- 5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
770	88- 6491	6547	6604	666	6716	6773	6829	6885	6942	6998	56
771	88- 7054	7111	7167	7223	728	7336	7392	7449	7505	7561	56
772	88- 7617	7674	773	7786	7842	7898	7955	8011	8067	8123	56
773	88- 8179	8236	8292	8348	8404	846	8516	8573	8629	8685	56
774	88- 8741	8797	8853	8909	8965	9021	9077	9134	919	9246	56
775	88- 9302	9358	9414	947	9526	9582	9638	9694	975	9806	56
776	88- 9862	9918	9974	—	—	—	—	—	—	—	56
776	89- —	—	—	003	0086	0141	0197	0253	0309	0365	56
777	89- 0421	0477	0533	0589	0645	07	0756	0812	0868	0924	56
778	89- 098	1035	1091	1147	1203	1259	1314	137	1426	1482	56
779	89- 1537	1593	1649	1705	176	1816	1872	1928	1983	2039	56
780	89- 2095	215	2206	2262	2317	2373	2429	2484	254	2595	56
781	89- 2651	2707	2762	2818	2873	2929	2985	304	3096	3151	56
782	89- 3207	3262	3318	3373	3429	3484	354	3595	3651	3706	56
783	89- 3762	3817	3873	3928	3984	4039	4094	415	4205	4261	55
784	89- 4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
785	89- 487	4925	498	5036	5091	5146	5201	5257	5312	5367	55
786	89- 5423	5478	5533	5588	5644	5699	5754	5809	5864	592	55
787	89- 5975	603	6085	614	6195	6251	6306	6361	6416	6471	55
788	89- 6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	89- 7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	89- 7627	7682	7737	7792	7847	7902	7957	8012	8067	8122	55
791	89- 8176	8231	8286	8341	8396	8451	8506	8561	8615	867	55
792	89- 8725	878	8835	889	8944	8999	9054	9109	9164	9218	55
793	89- 9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
794	89- 9821	9875	993	9985	—	—	—	—	—	—	55
794	90- —	—	—	—	0039	0094	0149	0203	0258	0312	55
795	90- 0367	0422	0476	0531	0586	064	0695	0749	0804	0859	55
796	90- 0913	0968	1022	1077	1131	1186	124	1295	1349	1404	55
797	90- 1458	1513	1567	1622	1676	1731	1785	184	1894	1948	54
798	90- 2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
799	90- 2547	2601	2655	271	2764	2818	2873	2927	2981	3036	54
800	90- 309	3144	3199	3253	3307	3361	3416	347	3524	3578	54
801	90- 3633	3687	3741	3795	3849	3904	3958	4012	4066	412	54
802	90- 4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
803	90- 4716	477	4824	4878	4932	4986	504	5094	5148	5202	54
804	90- 5256	531	5364	5418	5472	5526	558	5634	5688	5742	54
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805	90-	5796	585	5904	5958	6012	6066	6119	6173	6227	6281	54
806	90-	6335	6389	6443	6497	6551	6604	6658	6712	6766	682	54
807	90-	6874	6927	6981	7035	7089	7143	7196	725	7304	7358	54
808	90-	7411	7465	7519	7573	7626	768	7734	7787	7841	7895	54
809	90-	7949	8002	8056	811	8163	8217	827	8324	8378	8431	54
810	90-	8485	8539	8592	8646	8699	8753	8807	886	8914	8967	54
811	90-	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
812	90-	9556	9609	9663	9716	977	9823	9877	993	9984	—	54
812	91-	—	—	—	—	—	—	—	—	—	0037	53
813	91-	0091	0144	0197	0251	0304	0358	0411	0464	0518	0571	53
814	91-	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	91-	1158	1211	1264	1317	1371	1424	1477	153	1584	1637	53
816	91-	169	1743	1797	185	1903	1956	2009	2063	2116	2169	53
817	91-	2222	2275	2328	2381	2435	2488	2541	2594	2647	27	53
818	91-	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
819	91-	3284	3337	339	3443	3496	3549	3602	3655	3708	3761	53
820	91-	3814	3867	392	3973	4026	4079	4132	4184	4237	429	53
821	91-	4343	4396	4449	4502	4555	4608	466	4713	4766	4819	53
822	91-	4872	4925	4977	503	5083	5136	5189	5241	5294	5347	53
823	91-	54	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
824	91-	5927	598	6033	6085	6138	6191	6243	6296	6349	6401	53
825	91-	6454	6507	6559	6612	6664	6717	677	6822	6875	6927	53
826	91-	698	7033	7085	7138	719	7243	7295	7348	74	7453	53
827	91-	7506	7558	7611	7663	7716	7768	782	7873	7925	7978	52
828	91-	803	8083	8135	8188	824	8293	8345	8397	845	8502	52
829	91-	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
830	91-	9078	913	9183	9235	9287	934	9392	9444	9496	9549	52
831	91-	9601	9653	9706	9758	981	9862	9914	9967	—	—	52
831	92-	—	—	—	—	—	—	—	—	0019	0071	52
832	92-	0123	0176	0228	028	0332	0384	0436	0489	0541	0593	52
833	92-	0645	0697	0749	0801	0853	0906	0958	101	1062	1114	52
834	92-	1166	1218	127	1322	1374	1426	1478	153	1582	1634	52
835	92-	1686	1738	179	1842	1894	1946	1998	205	2102	2154	52
836	92-	2206	2258	231	2362	2414	2466	2518	257	2622	2674	52
837	92-	2725	2777	2829	2881	2933	2985	3037	3089	314	3192	52
838	92-	3244	3296	3348	3399	3451	3503	3555	3607	3658	371	52
839	92-	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
840	92-	4279	4331	4383	4434	4486	4538	4589	4641	4693	4744	52
841	92-	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
842	92-	5312	5364	5415	5467	5518	557	5621	5673	5725	5776	52
843	92-	5828	5879	5931	5982	6034	6085	6137	6188	624	6291	51
844	92-	6342	6394	6445	6497	6548	66	6651	6702	6754	6805	51
845	92-	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
846	92-	737	7422	7473	7524	7576	7627	7678	773	7781	7832	51
847	92-	7883	7935	7986	8037	8088	814	8191	8242	8293	8345	51
848	92-	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
849	92-	8908	8959	901	9061	9112	9163	9215	9266	9317	9368	51
850	92-	9419	947	9521	9572	9623	9674	9725	9776	9827	9879	51
851	92-	993	9981	—	—	—	—	—	—	—	—	51
851	93-	—	—	0032	0083	0134	0185	0236	0287	0338	0389	51
852	93-	044	0491	0542	0592	0643	0694	0745	0796	0847	0898	51
853	93-	0949	1	1051	1102	1153	1203	1254	1305	1356	1407	51
854	93-	1458	1509	156	161	1661	1712	1763	1814	1865	1915	51
No.		0	1	2	3	4	5	6	7	8	9	D

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855	93-	1966	2017	2068	2118	2169	222	2271	2322	2372	2423	51
856	93-	2474	2524	2575	2626	2677	2727	2778	2829	2879	293	51
857	93-	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
858	93-	3487	3538	3589	3639	369	374	3791	3841	3892	3943	51
859	93	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
860	93-	4498	4549	4599	465	47	4751	4801	4852	4902	4953	50
861	93-	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
862	93-	5507	5558	5608	5658	5709	5759	5809	586	591	596	50
863	93-	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
864	93-	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
865	93-	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
866	93-	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
867	93-	8019	8069	8119	8169	8219	8269	8319	837	842	847	50
868	93-	852	857	862	867	872	877	882	887	892	897	50
869	93-	902	907	912	917	922	927	932	9369	9419	9469	50
870	93-	9519	9569	9619	9669	9719	9769	9819	9869	9918	9968	50
871	94-	0018	0068	0118	0168	0218	0267	0317	0367	0417	0467	50
872	94-	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
873	94-	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
874	94-	1511	1561	1611	166	171	176	1809	1859	1909	1958	50
875	94-	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	94-	2504	2554	2603	2653	2702	2752	2801	2851	2901	295	50
877	94-	3	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	94-	3495	3544	3593	3643	3692	3742	3791	3841	389	3939	49
879	94-	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49
880	94-	4483	4532	4581	4631	468	4729	4779	4828	4877	4927	49
881	94-	4976	5025	5074	5124	5173	5222	5272	5321	537	5419	49
882	94-	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	94-	5961	601	6059	6108	6157	6207	6256	6305	6354	6403	49
884	94-	6452	6501	6551	66	6649	6698	6747	6796	6845	6894	49
885	94-	6943	6992	7041	709	714	7189	7238	7287	7336	7385	49
886	94-	7434	7483	7532	7581	763	7679	7728	7777	7826	7875	49
887	94-	7924	7973	8022	807	8119	8168	8217	8266	8315	8364	49
888	94-	8413	8462	8511	856	8609	8657	8706	8755	8804	8853	49
889	94-	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	94-	939	9439	9488	9536	9585	9634	9683	9731	978	9829	49
891	94-	9878	9926	9975	—	—	—	—	—	—	—	49
891	95-	—	—	—	0024	0073	0121	017	0219	0267	0316	49
892	95-	0365	0414	0462	0511	056	0608	0657	0706	0754	0803	49
893	95-	0851	09	0949	0997	1046	1095	1143	1192	124	1289	49
894	95-	1338	1386	1435	1483	1532	158	1629	1677	1726	1775	49
895	95-	1823	1872	192	1969	2017	2066	2114	2163	2211	226	48
896	95-	2308	2356	2405	2453	2502	255	2599	2647	2696	2744	48
897	95-	2792	2841	2889	2938	2986	3034	3083	3131	318	3228	48
898	95-	3276	3325	3373	3421	347	3518	3566	3615	3663	3711	48
899	95-	376	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
900	95-	4243	4291	4339	4387	4435	4484	4532	458	4628	4677	48
901	95-	4725	4773	4821	4869	4918	4966	5014	5062	511	5158	48
902	95-	5207	5255	5303	5351	5399	5447	5495	5543	5592	564	48
903	95-	5688	5736	5784	5832	588	5928	5976	6024	6072	612	48
904	95-	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
No.		0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
905	95-	6649	6697	6745	6793	684	6888	6936	6984	7032	708	48
906	95-	7128	7176	7224	7272	732	7368	7416	7464	7512	7559	48
907	95-	7607	7655	7703	7751	7799	7847	7894	7942	799	8038	48
908	95-	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
909	95-	8564	8612	8659	8707	8755	8803	885	8898	8946	8994	48
910	95-	9041	9089	9137	9185	9232	928	9328	9375	9423	9471	48
911	95-	9518	9566	9614	9661	9709	9757	9804	9852	99	9947	48
912	95-	9995	—	—	—	—	—	—	—	—	—	48
912	96-	—	0042	009	0138	0185	0233	028	0328	0376	0423	48
913	96-	0471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
914	96-	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
915	96-	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
916	96-	1895	1943	199	2038	2085	2132	218	2227	2275	2322	47
917	96-	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
918	96-	2843	289	2937	2985	3032	3079	3126	3174	3221	3268	47
919	96-	3316	3363	341	3457	3504	3552	3599	3646	3693	3741	47
920	96-	3788	3835	3882	3929	3977	4024	4071	4118	4165	4212	47
921	96-	426	4307	4354	4401	4448	4495	4542	459	4637	4684	47
922	96-	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
923	96-	5202	5249	5296	5343	539	5437	5484	5531	5578	5625	47
924	96-	5672	5719	5766	5813	586	5907	5954	6001	6048	6095	47
925	96-	6142	6189	6236	6283	6329	6376	6423	647	6517	6564	47
926	96-	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	96-	708	7127	7173	722	7267	7314	7361	7408	7454	7501	47
928	96-	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
929	96-	8016	8062	8109	8156	8203	8249	8296	8343	839	8436	47
930	96-	8483	853	8576	8623	867	8716	8763	881	8856	8903	47
931	96-	895	8996	9043	909	9136	9183	9229	9276	9323	9369	47
932	96-	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
933	96-	9882	9928	9975	—	—	—	—	—	—	—	47
933	97-	—	—	—	0021	0068	0114	0161	0207	0254	03	47
934	97-	0347	0393	044	0486	0533	0579	0626	0672	0719	0765	46
935	97-	0812	0858	0904	0951	0997	1044	109	1137	1183	1229	46
936	97-	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	97-	174	1786	1832	1879	1925	1971	2018	2064	211	2157	46
938	97-	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
939	97-	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46
940	97-	3128	3174	322	3266	3313	3359	3405	3451	3497	3543	46
941	97-	359	3636	3682	3728	3774	382	3866	3913	3959	4005	46
942	97-	4051	4097	4143	4189	4235	4281	4327	4374	442	4466	46
943	97-	4512	4558	4604	465	4696	4742	4788	4834	488	4926	46
944	97-	4972	5018	5064	511	5156	5202	5248	5294	534	5386	46
945	97-	5432	5478	5524	557	5616	5662	5707	5753	5799	5845	46
946	97-	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
947	97-	635	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
948	97-	6808	6854	69	6946	6992	7037	7083	7129	7175	722	46
949	97-	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
950	97-	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	46
951	97-	8181	8226	8272	8317	8363	8409	8454	85	8546	8591	46
952	97-	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
953	97-	9093	9138	9184	923	9275	9321	9366	9412	9457	9503	46
954	97-	9548	9594	9639	9685	973	9776	9821	9867	9912	9958	46
No.		0	1	2	3	4	5	6	7	8	9	D

No.		0	1	2	3	4	5	6	7	8	9	D
955	98-	0003	0049	0094	014	0185	0231	0276	0322	0367	0412	45
956	98-	0458	0503	0549	0594	064	0685	073	0776	0821	0867	45
957	98-	0912	0957	1003	1048	1093	1139	1184	1229	1275	132	45
958	98-	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
959	98-	1819	1864	1909	1954	2	2045	209	2135	2181	2226	45
960	98-	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	45
961	98-	2723	2769	2814	2859	2904	2949	2994	304	3085	313	45
962	98-	3175	322	3265	331	3356	3401	3446	3491	3536	3581	45
963	98-	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
964	98-	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
965	98-	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
966	98-	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
967	98-	5426	5471	5516	5561	5606	5651	5696	5741	5786	583	45
968	98-	5875	592	5965	601	6055	61	6144	6189	6234	6279	45
969	98-	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
970	98-	6772	6817	6861	6906	6951	6996	704	7085	713	7175	45
971	98-	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
972	98-	7666	7711	7756	78	7845	789	7934	7979	8024	8068	45
973	98-	8113	8157	8202	8247	8291	8336	8381	8425	847	8514	45
974	98-	8559	8604	8648	8693	8737	8782	8826	8871	8916	896	45
975	98-	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	98-	945	9494	9539	9583	9628	9672	9717	9761	9806	985	44
977	98-	9895	9939	9983	—	—	—	—	—	—	—	44
977	99-	—	—	—	0028	0072	0117	0161	0206	025	0294	44
978	99-	0339	0383	0428	0472	0516	0561	0605	065	0694	0738	44
979	99-	0783	0827	0871	0916	096	1004	1049	1093	1137	1182	44
980	99-	1226	127	1315	1359	1403	1448	1492	1536	158	1625	44
981	99-	1669	1713	1758	1802	1846	189	1935	1979	2023	2067	44
982	99-	2111	2156	22	2244	2288	2333	2377	2421	2465	2509	44
983	99-	2554	2598	2642	2686	273	2774	2819	2863	2907	2951	44
984	99-	2995	3039	3083	3127	3172	3216	326	3304	3348	3392	44
985	99-	3436	348	3524	3568	3613	3657	3701	3745	3789	3833	44
986	99-	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	99-	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	99-	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	99-	5196	524	5284	5328	5372	5416	546	5504	5547	5591	44
990	99-	5635	5679	5723	5767	5811	5854	5898	5942	5986	603	44
991	99-	6074	6117	6161	6205	6249	6293	6337	638	6424	6468	44
992	99-	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	99-	6949	6993	7037	708	7124	7168	7212	7255	7299	7343	44
994	99-	7386	743	7474	7517	7561	7605	7648	7692	7736	7779	44
995	99-	7823	7867	791	7954	7998	8041	8085	8129	8172	8216	44
996	99-	8259	8303	8347	839	8434	8477	8521	8564	8608	8652	44
997	99-	8695	8739	8782	8826	8869	8913	8956	9	9043	9087	44
998	99-	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	99-	9565	9609	9652	9696	9739	9783	9826	987	9913	9957	43
No.		0	1	2	3	4	5	6	7	8	9	D

NATURAL SINES AND COSINES.

Prop. parts		0°		1°		2°		3°			Prop. parts
29	'	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		2
0	0	.00000	1	01745	.99985	0349	.99939	.05234	.99863	60	2
0	1	.00029	1	.01774	.99984	.03519	.99938	.05263	.99861	59	2
1	2	.00058	1	01803	.99984	.03548	.99927	.05292	.9986	58	2
1	3	.00087	1	.01832	.99983	.03577	.99936	.05321	.99858	57	2
2	4	.00116	1	.01862	.99983	03606	.99935	.0535	.99857	56	2
2	5	.00145	1	.01891	.99982	03635	.99934	.05379	.99855	55	2
3	6	.00175	1	.0192	.99982	.03664	.99933	.05408	.99854	54	2
3	7	.00204	1	.01949	.99981	03693	.99932	.05437	.99852	53	2
4	8	.00233	1	.01978	.9998	.03723	.99931	.05466	.99851	52	2
4	9	.00262	1	.02007	.9998	.03752	.9993	.05495	.99849	51	2
5	10	.00291	1	.02036	.99979	03781	.99929	.05524	.99847	50	2
5	11	.0032	.99999	02065	.99979	.0381	.99927	.05553	.99846	49	2
6	12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	48	2
6	13	.00378	.99999	.02123	.99977	03868	.99925	.05611	.99842	47	2
7	14	.00407	.99999	.02152	.99977	.03897	.99924	.0564	.99841	46	2
7	15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	45	2
8	16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	44	1
8	17	.00495	.99999	.0224	.99975	.03984	.99921	.05727	.99836	43	1
9	18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	42	1
9	19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	41	1
10	20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	40	1
10	21	.00611	.99998	.02356	.99972	.041	.99916	.05844	.99829	39	1
11	22	.0064	.99998	.02385	.99972	.04129	99915	.05873	99827	38	1
11	23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	37	1
12	24	.00698	.99998	.02443	.9997	.04188	.99912	.05931	.99824	36	1
12	25	.00727	.99997	.02472	.99969	.04217	.99911	.0596	.99822	35	1
13	26	.00756	.99997	.02501	.99969	.04246	.9991	.05989	.99821	34	1
13	27	.00785	.99997	.0253	.99968	.04275	.99909	.06018	.99819	33	1
14	28	.00814	.99997	.0256	.99967	.04304	.99907	.06047	.99817	32	1
14	29	.00844	.99996	.02589	.99966	.04333	99906	.06076	.99815	31	1
15	30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	30	1
15	31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	99812	29	1
15	32	.00931	.99996	.02676	.99964	.0442	.99902	.06163	.9981	28	1
16	33	.0096	.99995	.02705	.99963	.04449	.99901	.06192	.99808	27	1
16	34	.00989	.99995	.02734	.99963	.04478	.999	.06221	.99806	26	1
17	35	01018	.99995	.02763	.99962	.04507	.99898	.0625	.99804	25	1
17	36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	24	1
18	37	.01076	.99994	.02821	.9996	.04565	.99896	.06308	.99801	23	1
18	38	.01105	.99994	0285	.99959	.04594	.99894	.06337	.99799	22	1
19	39	.01134	.99994	.02879	.99959	.04623	99893	.06366	.99797	21	1
19	40	.01164	.99993	.02908	.99958	.04653	99892	.06395	.99795	20	1
20	41	.01193	.99993	.02938	.99957	.04682	.9989	.06424	.99793	19	1
20	42	.01222	.99993	.02967	.99956	.04711	.99889	06453	.99792	18	1
21	43	.01251	.99992	02996	.99955	.0474	.99888	.06482	9979	17	1
21	44	.0128	.99992	.03025	.99954	.04769	.99886	.06511	.99788	16	1
22	45	01309	.99991	.03054	.99953	.04798	.99885	.0654	.99786	15	1
22	46	.01338	.99991	.03083	.99952	.04827	.99883	06569	.99784	14	0
23	47	.01367	.99991	.03112	.99952	.04856	.99882	06598	.99782	13	0
23	48	.01396	.9999	.03141	.99951	04885	.99881	.06627	.9978	12	0
24	49	.01425	.9999	.0317	.9995	04914	99879	.06656	.99778	11	0
24	50	.01454	.99989	.03199	.99949	04943	.99878	06685	.99776	10	0
25	51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	9	0
25	52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	8	0
26	53	.01542	.99988	.03286	.99946	.0503	.99873	.06773	.9977	7	0
26	54	.01571	.99988	.03316	.99945	05059	.99872	.06802	.99768	6	0
27	55	.016	.99987	.03345	.99944	.05088	.9987	.06831	.99766	5	0
27	56	.01629	.99987	.03374	.99943	.05117	.99869	.0686	.99764	4	0
28	57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	3	0
28	58	.01687	.99986	.03432	.99941	05175	.99866	.06918	.9976	2	0
29	59	.01716	.99985	.03461	.9994	.05205	.99864	.06947	.99758	1	0
29	60	.01745	.99985	.0349	.99939	.05234	.99863	.06976	.99756	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	'	
		89°		88°		87°		86°			

Prop. parts	′	4°		5°		6°		7°			Prop. parts
29		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		4
0	0	.06976	.99756	08716	.99619	.10453	.99452	.12187	.99255	60	4
0	1	.07005	99754	.08745	.99617	.10482	.99449	.12216	.99251	59	4
1	2	.07034	.99752	.08774	.99614	.10511	.99446	.12245	.99248	58	4
1	3	.07063	.9975	08803	.99612	.1054	.99443	12274	.99244	57	4
2	4	.07092	.99748	.08831	.99609	.10569	.9944	.12302	.9924	56	4
2	5	.07121	.99746	.0886	.99607	.10597	.99437	12331	.99237	55	4
3	6	.0715	.99744	.08889	.99604	.10626	.99434	1236	.99233	54	4
3	7	.07179	.99742	.08918	.99602	.10655	.99431	12389	.9923	53	4
4	8	.07208	.9974	.08947	.99599	.10684	.99428	12418	.99226	52	3
4	9	.07237	.99738	.08976	.99596	.10713	.99424	.12447	.99222	51	3
5	10	.07266	.99736	.09005	.99594	.10742	.99421	.12476	.99219	50	3
5	11	.07295	.99734	.09034	.99591	.10771	.99418	.12504	.99215	49	3
6	12	.07324	.99731	.09063	.99588	.108	.99415	.12533	.99211	48	3
6	13	.07353	.99729	.09092	.99586	.10829	.99412	12562	.99208	47	3
7	14	.07382	.99727	.09121	.99583	10858	.99409	.12591	.99204	46	3
7	15	.07411	.99725	.0915	.9958	.10887	.99406	.1262	.992	45	3
8	16	.0744	.99723	.09179	.99578	.10916	.99402	.12649	.99197	44	3
8	17	.07469	.99721	.09208	.99575	.10945	.99399	.12678	.99193	43	3
9	18	.07498	.99719	.09237	.99572	.10973	.99396	.12706	.99189	42	3
9	19	.07527	.99716	.09266	.9957	.11002	.99393	.12735	.99186	41	3
10	20	.07556	.99714	.09295	.99567	11031	.9939	.12764	.99182	40	3
10	21	.07585	.99712	.09324	.99564	.1106	.99386	.12793	.99178	39	3
11	22	.07614	.9971	.09353	.99562	.11089	.99383	.12822	.99175	38	3
11	23	.07643	.99708	.09382	.99559	.11118	.9938	12851	.99171	37	2
12	24	.07672	.99705	.09411	.99556	.11147	.99377	.1288	.99167	36	2
12	25	.07701	.99703	.0944	.99553	11176	.99374	.12908	99163	35	2
13	26	.0773	99701	09469	.99551	.11205	.9937	.12937	.9916	34	2
13	27	.07759	.99699	.09498	.99548	11234	.99367	.12966	.99156	33	2
14	28	.07788	.99696	.09527	.99545	.11263	.99364	.12995	.99152	32	2
14	29	.07817	.99694	.09556	.99542	.11291	.9936	.13024	.99148	31	2
15	30	.07846	.99692	.09585	.9954	.1132	.99357	13053	.99144	30	2
15	31	.07875	.99689	.09614	.99537	.11349	.99354	13081	.99141	29	2
15	32	.07904	.99687	.09642	.99534	.11378	.99351	.1311	.99137	28	2
16	33	.07933	.99685	.09671	.99531	.11407	.99347	.13139	.99133	27	2
16	34	.07962	.99683	.097	.99528	.11436	.99344	.13168	.99129	26	2
17	35	.07991	.9968	.09729	.99526	.11465	.99341	.13197	.99125	25	2
17	36	.0802	.99678	.09758	.99523	.11494	.99337	.13226	.99122	24	2
18	37	.08049	.99676	.09787	.9952	.11523	.99334	.13254	.99118	23	2
18	38	.08078	.99673	.09816	.99517	.11552	.99331	.13283	.99114	22	1
19	39	.08107	.99671	.09845	.99514	.1158	.99327	.13312	.9911	21	1
19	40	.08136	.99668	.09874	.99511	11609	.99324	.13341	.99106	20	1
20	41	.08165	.99666	.09903	.99508	.11638	.9932	.1337	.99102	19	1
20	42	.08194	.99664	.09932	.99506	.11667	.99317	13399	.99098	18	1
21	43	.08223	.99661	.09961	.99503	.11696	.99314	13427	.99094	17	1
21	44	.08252	.99659	.0999	.995	.11725	.9931	.13456	.99091	16	1
22	45	.08281	.99657	.10019	.99497	.11754	.99307	.13485	.99087	15	1
22	46	.0831	.99654	.10048	.99494	11783	.99303	.13514	.99083	14	1
23	47	.08339	.99652	.10077	.99491	.11812	993	.13543	.99079	13	1
23	48	.08368	.99649	.10106	.99488	.1184	.99297	.13572	.99075	12	1
24	49	.08397	.99647	.10135	.99485	.11869	.99293	.136	.99071	11	1
24	50	.08426	.99644	.10164	.99482	.11898	.9929	.13629	.99067	10	1
25	51	.08455	.99642	.10192	.99479	.11927	99286	.13658	.99063	9	1
25	52	.08484	.99639	.10221	.99476	.11956	.99283	.13687	99059	8	1
26	53	.08513	.99637	.1025	.99473	.11985	.99279	.13716	.99055	7	0
26	54	.08542	.99635	.10279	.9947	.12014	.99276	.13744	.99051	6	0
27	55	.08571	.99632	.10308	.99467	.12043	.99272	.13773	.99047	5	0
27	56	.086	.9963	.10337	.99464	.12071	.99269	.13802	.99043	4	0
28	57	.08629	.99627	10366	.99461	.121	.99265	.13831	.99039	3	0
28	58	.08658	.99625	.10395	.99458	12129	.99262	.1386	.99035	2	0
29	59	.08687	.99622	10424	.99455	.12158	.99258	.13889	99031	1	0
29	60	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		85°		84°		83°		82°			

Prop. parts. 28	′	8°		9°		10°		11°			Prop. parts. 6
		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		
0	0	.13917	.99027	.15643	.98769	.17365	.98481	.19081	.98163	60	6
0	1	.13946	.99023	.15672	.98764	.17393	.98476	.19109	.98157	59	6
1	2	.13975	.99019	.15701	.9876	.17422	.98471	.19138	.98152	58	6
1	3	.14004	.99015	.1573	.98755	.17451	.98466	.19167	.98146	57	6
2	4	.14033	.99011	.15758	.98751	.17479	.98461	.19195	.9814	56	6
2	5	.14061	.99006	.15787	.98746	.17508	.98455	.19224	.98135	55	6
3	6	.1409	.99002	.15816	.98741	.17537	.9845	.19252	.98129	54	5
3	7	.14119	.98998	.15845	.98737	.17565	.98445	.19281	.98124	53	5
4	8	.14148	.98994	.15873	.98732	.17594	.9844	.19309	.98118	52	5
4	9	.14177	.9899	.15902	.98728	.17623	.98435	.19338	.98112	51	5
5	10	.14205	.98986	.15931	.98723	.17651	.9843	.19366	.98107	50	5
5	11	.14234	.98982	.15959	.98718	.1768	.98425	.19395	.98101	49	5
6	12	.14263	.98978	.15988	.98714	.17708	.9842	.19423	.98096	48	5
6	13	.14292	.98973	.16017	.98709	.17737	.98414	.19452	.9809	47	5
7	14	.1432	.98969	.16046	.98704	.17766	.98409	.19481	.98084	46	5
7	15	.14349	.98965	.16074	.987	.17794	.98404	.19509	.98079	45	5
7	16	14378	.98961	.16103	.98695	.17823	.98399	.19538	.98073	44	4
8	17	.14407	.98957	.16132	.9869	.17852	.98394	.19566	.98067	43	4
8	18	.14436	.98953	.1616	.98686	.1788	.98389	.19595	.98061	42	4
9	19	.14464	.98948	.16189	.98681	.17909	.98383	.19623	.98056	41	4
9	20	.14493	.98944	.16218	.98676	.17937	.98378	.19652	.9805	40	4
10	21	.14522	.9894	.16246	.98671	.17966	.98373	.1968	.98044	39	4
10	22	.14551	.98936	.16275	.98667	.17995	.98368	.19709	.98039	38	4
11	23	.1458	.98931	.16304	.98662	.18023	.98362	.19737	.98033	37	4
11	24	.14608	.98927	.16333	.98657	.18052	.98357	.19766	.98027	36	4
12	25	.14637	.98923	.16361	.98652	.18081	.98352	.19794	.98021	35	4
12	26	.14666	.98919	.1639	.98648	.18109	.98347	.19823	.98016	34	3
13	27	.14695	.98914	.16419	.98643	.18138	.98341	.19851	.9801	33	3
13	28	.14723	.9891	.16447	.98638	.18166	.98336	.1988	.98004	32	3
14	29	.14752	.98906	.16476	.98633	.18195	.98331	.19908	.97988	31	3
14	30	.14781	.98902	.16505	.98629	.18224	.98325	.19937	.97992	30	3
14	31	.1481	.98897	.16533	.98624	.18252	.9832	.19965	.97987	29	3
15	32	.14838	.98893	.16562	.98619	.18281	.98315	.19994	.97981	28	3
15	33	.14867	.98889	.16591	.98614	.18309	.9831	.20022	.97975	27	3
16	34	.14896	.98884	.1662	.98609	.18338	.98304	.20051	.97969	26	3
16	35	.14925	.9888	.16648	.98604	.18367	.98299	.20079	.97963	25	3
17	36	.14954	.98876	.16677	.986	.18395	.98294	.20108	.97958	24	2
17	37	.14982	.98871	.16706	.98595	.18424	.98288	.20136	.97952	23	2
18	38	.15011	.98867	.16734	.9859	.18452	.98283	.20165	.97946	22	2
18	39	.1504	.98863	.16763	.98585	.18481	.98277	.20193	.9794	21	2
19	40	.15069	.98858	.16792	.9858	.18509	.98272	.20222	.97934	20	2
19	41	.15097	.98854	.1682	.98575	.18538	.98267	.2025	.97928	19	2
20	42	.15126	.98849	.16849	.9857	.18567	.98261	.20279	97922	18	2
20	43	15155	.98845	.16878	.98565	18595	.98256	.20307	.97916	17	2
21	44	.15184	.98841	.16906	.98561	.18624	.9825	.20336	.9791	16	2
21	45	.15212	.98836	.16935	.98556	.18652	.98245	.20364	.97905	15	2
21	46	.15241	.98832	.16964	.98551	.18681	.9824	.20393	.97899	14	1
22	47	.1527	.98827	.16992	.98546	.1871	.98234	.20421	.97893	13	1
22	48	.15299	.98823	.17021	.98541	.18738	.98229	.2045	97887	12	1
23	49	.15327	.98818	.1705	.98536	.18767	.98223	.20478	97881	11	1
23	50	.15356	.98814	17078	.98531	.18795	.98218	.20507	.97875	10	1
24	51	.15385	.98809	.17107	.98526	18824	.98212	.20535	.97869	9	1
24	52	.15414	.98805	.17136	.98521	.18852	.98207	.20563	.97863	8	1
25	53	.15442	.988	17164	.98516	18881	.98201	.20592	.97857	7	1
25	54	.15471	.98796	.17193	.98511	.1891	.98196	.2062	.97851	6	1
26	55	.155	.98791	.17222	.98506	.18938	.9819	.20649	.97845	5	1
26	56	.15529	.98787	.1725	.98501	.18967	.98185	.20677	.97839	4	0
27	57	.15557	.98782	.17279	.98496	.18995	.98179	.20706	.97833	3	0
27	58	.15586	.98778	17308	.98491	.19024	.98174	.20734	.97827	2	0
28	59	.15615	.98773	17336	.98486	.19052	.98168	.20763	.97821	1	0
28	60	.15643	98769	17365	.98481	.19081	.98163	.20791	.97815	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		81°		80°		79°		78°			

Prop. parts		12°		13°		14°		15°			Prop. parts
27		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		9
0	0	.20791	.97815	.22495	.97437	.24192	.9703	.25882	.96593	60	9
0	1	.2082	.97809	.22523	.9743	.2422	.97023	.2591	.96585	59	9
1	2	.20848	.97803	.22552	.97424	.24249	.97015	.25938	.96578	58	9
1	3	.20877	.97797	.2258	.97417	.24277	.97008	.25966	.9657	57	9
2	4	.20905	.97791	.22608	.97411	.24305	.97001	.25994	.96562	56	8
2	5	.20933	.97784	.22637	.97404	.24333	.96994	.26022	.96555	55	8
3	6	.20962	.97778	.22665	.97398	.24362	.96987	.2605	.96547	54	8
3	7	.2099	.97772	.22693	.97391	.2439	.9698	.26079	.9654	53	8
4	8	.21019	.97766	.22722	.97384	.24418	.96973	.26107	.96532	52	8
4	9	.21047	.9776	.2275	.97378	.24446	.96966	.26135	.96524	51	8
5	10	.21076	.97754	.22778	.97371	.24474	.96959	.26163	.96517	50	8
5	11	.21104	.97748	.22807	.97365	.24503	.96952	.26191	.96509	49	7
5	12	.21132	.97742	.22835	.97358	.24531	.96945	.26219	.96502	48	7
6	13	.21161	.97735	.22863	.97351	.24559	.96937	.26247	.96494	47	7
6	14	.21189	.97729	.22892	.97345	.24587	.9693	.26275	.96486	46	7
7	15	.21218	.97723	.2292	.97338	.24615	.96923	.26303	.96479	45	7
7	16	.21246	.97717	.22948	.97331	.24644	.96916	.26331	.96471	44	7
8	17	.21275	.97711	.22977	.97325	.24672	.96909	.26359	.96463	43	6
8	18	.21303	.97705	.23005	.97318	.247	.96902	.26387	.96456	42	6
9	19	.21331	.97698	.23033	.97311	.24728	.96894	.26415	.96448	41	6
9	20	.2136	.97692	.23062	.97304	.24756	.96887	.26443	.9644	40	6
9	21	.21388	.97686	.2309	.97298	.24784	.9688	.26471	.96433	39	6
10	22	.21417	.9768	.23118	.97291	.24813	.96873	.265	.96425	38	6
10	23	.21445	.97673	.23146	.97284	.24841	.96866	.26528	.96417	37	6
11	24	.21474	.97667	.23175	.97278	.24869	.96858	.26556	.9641	36	5
11	25	.21502	.97661	.23203	.97271	.24897	.96851	.26584	.96402	35	5
12	26	.2153	.97655	.23231	.97264	.24925	.96844	.26612	.96394	34	5
12	27	.21559	.97648	.2326	.97257	.24954	.96837	.2664	.96386	33	5
13	28	.21587	.97642	.23288	.97251	.24982	.96829	.26668	.96379	32	5
13	29	.21616	.97636	.23316	.97244	.2501	.96822	.26696	.96371	31	5
14	30	.21644	.9763	.23345	97237	.25038	.96815	.26724	.96363	30	5
14	31	.21672	.97623	.23373	.9723	.25066	.96807	.26752	.96355	29	4
14	32	.21701	.97617	.23401	.97223	.25094	.968	.2678	.96347	28	4
15	33	.21729	.97611	.23429	.97217	.25122	.96793	.26808	.9634	27	4
15	34	.21758	.97604	.23458	.9721	.25151	.96786	.26836	.96332	26	4
16	35	.21786	.97598	.23486	.97203	.25179	.96778	.26864	.96324	25	4
16	36	.21814	.97592	.23514	.97196	.25207	.96771	.26892	.96316	24	4
17	37	.21843	.97585	.23542	.97189	.25235	.96764	.2692	.96308	23	3
17	38	.21871	.97579	.23571	.97182	.25263	.96756	.26948	.96301	22	3
18	39	.21899	.97573	.23599	.97176	.25291	.96749	.26976	.96293	21	3
18	40	.21928	.97566	.23627	.97169	.2532	.96742	.27004	.96285	20	3
18	41	.21956	.9756	.23656	.97162	.25348	.96734	.27032	.96277	19	3
19	42	.21985	.97553	.23684	.97155	.25376	.96727	.2706	.96269	18	3
19	43	.22013	.97547	.23712	.97148	.25404	.96719	.27088	.96261	17	3
20	44	.22041	.97541	.2374	.97141	.25432	.96712	.27116	.96253	16	2
20	45	.2207	.97534	.23769	.97134	.2546	.96705	.27144	.96246	15	2
21	46	.22098	.97528	.23797	.97127	.25488	.96697	.27172	.96238	14	2
21	47	.22126	.97521	.23825	.9712	.25516	.9669	.272	.9623	13	2
22	48	.22155	.97515	.23853	.97113	.25545	.96682	.27228	.96222	12	2
22	49	.22183	.97508	.23882	.97106	.25573	.96675	.27256	.96214	11	2
23	50	.22212	.97502	.2391	.971	.25601	.96667	.27284	.96206	10	2
23	51	.2224	.97496	.23938	.97093	.25629	.9666	.27312	.96198	9	1
23	52	.22268	97489	.23966	.97086	.25657	.96653	.2734	.9619	8	1
24	53	22297	97483	.23995	.97079	.25685	.96645	.27368	.96182	7	1
24	54	.22325	97476	.24023	.97072	.25713	.96638	.27396	.96174	6	1
25	55	.22353	9747	.24051	.97065	.25741	.9663	.27424	.96166	5	1
25	56	.22382	.97463	.24079	.97058	.25769	.96623	.27452	.96158	4	1
26	57	.2241	97457	.24108	.97051	.25798	.96615	.2748	.9615	3	0
26	58	.22438	9745	.24136	.97044	.25826	.96608	.27508	.96142	2	0
27	59	.22467	.97444	.24164	.97037	.25854	.966	.27536	.96134	1	0
27	60	.22495	.97437	.24192	.9703	.25882	.96593	.27564	.96126	0	
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.		
		77°		76°		75°		74°			

Prop. parts	′	16°		17°		18°		19°			Prop. parts
27		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		9
0	0	.27564	.96126	.29237	.9563	.30902	.95106	.32557	.94552	60	9
0	1	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59	9
1	2	.2762	.9611	.29293	.95613	.30957	.95088	.32612	.94533	58	9
1	3	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57	9
2	4	.27676	.96094	29348	.95596	.31012	.9507	.32667	.94514	56	8
2	5	.27704	.96086	.29376	.95588	.3104	.95061	.32694	.94504	55	8
3	6	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54	8
3	7	.27759	.9607	29432	.95571	.31095	.95043	.32749	.94485	53	8
4	8	.27787	.96062	.2946	.95562	.31123	.95033	.32777	.94476	52	8
4	9	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51	8
5	10	.27843	.96046	29515	.95545	.31178	.95015	.32832	.94457	50	8
5	11	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49	7
5	12	.27899	.96029	29571	.95528	.31233	.94997	.32887	.94438	48	7
6	13	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47	7
6	14	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46	7
7	15	.27983	.96005	29654	.95502	.31316	.9497	.32969	.94409	45	7
7	16	.28011	.95997	29682	95493	.31344	.94961	.32997	.94399	44	7
8	17	.28039	.95989	.2971	.95485	31372	.94952	.33024	.9439	43	6
8	18	.28067	95981	29737	.95476	.31399	.94943	.33051	.9438	42	6
9	19	.28095	95972	29765	95467	.31427	.94933	.33079	.9437	41	6
9	20	.28123	.95964	29793	.95459	.31454	.94924	.33106	.94361	40	6
9	21	.2815	.95956	29821	.9545	.31482	.94915	.33134	.94351	39	6
10	22	.28178	.95948	29849	.95441	3151	.94906	.33161	.94342	38	6
10	23	.28206	.9594	29876	95433	.31537	.94897	.33189	.94332	37	6
11	24	.28234	.95931	29904	95424	.31565	.94888	.33216	.94322	36	5
11	25	.28262	.95923	29932	.95415	31593	.94878	.33244	.94313	35	5
12	26	.2829	.95915	2996	.95407	.3162	.94869	.33271	.94303	34	5
12	27	.28318	.95907	.29987	95398	.31648	.9486	.33298	.94293	33	5
13	28	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32	5
13	29	.28374	.9589	.30043	.9538	.31703	.94842	.33353	.94274	31	5
14	30	.28402	.95882	.30071	.95372	.3173	.94832	.33381	.94264	30	5
14	31	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29	4
14	32	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28	4
15	33	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27	4
15	34	.28513	.95849	.30182	.95337	.31841	.94795	.3349	.94225	26	4
16	35	.28541	.95841	30209	.95328	.31868	.94786	.33518	.94215	25	4
16	36	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24	4
17	37	.28597	.95824	.30265	.9531	.31923	.94768	.33573	.94196	23	3
17	38	.28625	95816	.30292	.95301	.31951	.94758	.336	.94186	22	3
18	39	.28652	.95807	.3032	.95293	.31979	.94749	.33627	.94176	21	3
18	40	2868	.95799	.30348	.95284	.32006	.9474	.33655	.94167	20	3
18	41	.28708	.95791	.30376	.95275	.32034	.9473	.33682	.94157	19	3
19	42	.28736	.95782	.30403	.95266	.32061	.94721	.3371	.94147	18	3
19	43	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17	3
20	44	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16	2
20	45	.2882	.95757	.30486	.9524	.32144	.94693	.33792	.94118	15	2
21	46	.28847	.95749	.30514	95231	.32171	94684	.33819	.94108	14	2
21	47	.28875	.9574	30542	.95222	.32199	.94674	.33846	.94098	13	2
22	48	.28903	.95732	.3057	.95213	.32227	.94665	.33874	.94088	12	2
22	49	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11	2
23	50	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10	2
23	51	.28987	.95707	30653	.95186	.32309	.94637	.33956	.94058	9	1
23	52	.29015	.95698	.3068	.95177	.32337	.94627	.33983	.94049	8	1
24	53	.29042	.9569	.30708	.95168	.32364	.94618	.34011	.94039	7	1
24	54	.2907	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6	1
25	55	.29098	.95673	30763	.9515	.32419	.94599	.34065	.94019	5	1
25	56	.29126	.95664	.30791	.95142	.32447	.9459	.34093	.94009	4	1
26	57	.29154	.95656	.30819	.95133	.32474	.9458	.3412	.93999	3	0
26	58	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2	0
27	59	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1	0
27	60	.29237	.9563	.30902	.95106	.32557	.94552	.34202	.93969	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		73°		72°		71°		70°			

Prop. parts		20°		21°		22°		23°			Prop. parts
27		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		11
0	0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.9205	60	11
0	1	.34229	.93959	.35864	.93348	.37488	.92707	.391	.92039	59	11
1	2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	58	11
1	3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	57	10
2	4	.34311	.93929	.35945	.93316	.37569	.92675	.3918	.92005	56	10
2	5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	55	10
3	6	.34366	.93909	.36	.93295	.37622	.92653	.39234	.91982	54	10
3	7	.34393	.93899	.36027	.93285	.37649	.92642	.3926	.91971	53	10
4	8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	52	10
4	9	.34448	.93879	.36081	.93264	.37703	.9262	.39314	.91948	51	9
5	10	.34475	.93869	.36108	.93253	.3773	.92609	.39341	.91936	50	9
5	11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	49	9
5	12	.3453	.93849	.36162	.93232	.37784	.92587	.39394	.91914	48	9
6	13	.34557	.93839	.3619	.93222	.37811	.92576	.39421	.91902	47	9
6	14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	46	8
7	15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	45	8
7	16	.34639	.93809	.36271	.9319	.37892	.92543	.39501	.91868	44	8
8	17	.34066	.93799	.36298	.9318	.37919	.92532	.39528	.91856	43	8
8	18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	42	8
9	19	.34721	.93779	.36352	.93159	.37973	.9251	.39581	.91833	41	8
9	20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	40	7
9	21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.9181	39	7
10	22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	38	7
10	23	.3483	.93738	.36461	.93116	.3808	.92466	.39688	.91787	37	7
11	24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	36	7
11	25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	35	6
12	26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	34	6
12	27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	33	6
13	28	.34966	.93688	.36596	.93063	.38215	.9241	.39822	.91729	32	6
13	29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	31	6
14	30	.35021	.93667	.3665	.93042	.38268	.92388	.39875	.91706	30	6
14	31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	29	5
14	32	.35075	.93647	.36704	.9302	.38322	.92366	.39928	.91683	28	5
15	33	.35102	.93637	.36731	.9301	.38349	.92355	.39955	.91671	27	5
15	34	.3513	.93626	.36758	.92999	.38376	.92343	.39982	.9166	26	5
16	35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	25	5
16	36	.35184	.93606	.36812	.92978	.3843	.92321	.40035	.91636	24	4
17	37	.35211	.93596	.36839	.92967	.38456	.9231	.40062	.91625	23	4
17	38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	22	4
18	39	.35266	.93575	.36894	.92945	.3851	.92287	.40115	.91601	21	4
18	40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.9159	20	4
18	41	.3532	.93555	.36948	.92924	.38564	.92265	.40168	.91578	19	3
19	42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	18	3
19	43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	17	3
20	44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	16	3
20	45	.35429	.93514	.37056	.92881	.38671	.9222	.40275	.91531	15	3
21	46	.35456	.93503	.37083	.9287	.38698	.92209	.40301	.91519	14	3
21	47	.35484	.93493	.3711	.92859	.38725	.92198	.40328	.91508	13	2
22	48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	12	2
22	49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	11	2
23	50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	10	2
23	51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	9	2
23	52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	8	1
24	53	.35647	.93431	.37272	.92794	.38886	.9213	.40488	.91437	7	1
24	54	.35674	.9342	.37299	.92784	.38912	.92119	.40514	.91425	6	1
25	55	.35701	.9341	.37326	.92773	.38939	.92107	.40541	.91414	5	1
25	56	.35728	.934	.37353	.92762	.38966	.92096	.40567	.91402	4	1
26	57	.35755	.93389	.3738	.92751	.38993	.92085	.40594	.9139	3	[illegible]
26	58	.35782	.93379	.37407	.9274	.3902	.92073	.40621	.91378	2	0
27	59	.3581	.93368	.37434	.92729	.39046	.92062	.40647	.91366	1	0
27	60	.35837	.93358	.37461	.92718	.39073	.9205	.40674	.91355	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		69°		68°		67°		66°			

Prop. parts		24°		25°		26°		27°			Prop. parts
26	′	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		14
0	0	.40674	.91355	.42262	.90631	.43837	.89879	.45399	.89101	60	14
0	1	.407	.91343	.42288	.90618	.43863	.89867	.45425	89087	59	14
1	2	.40727	.91331	.42315	.90606	.43889	.89854	.45451	.89074	58	14
1	3	.40753	.91319	.42341	.90594	.43916	.89841	.45477	.89061	57	13
2	4	.4078	.91307	.42367	.90582	.43942	.89828	.45503	.89048	56	13
2	5	.40806	.91295	.42394	.90569	.43968	.89816	45529	.89035	55	13
3	6	.40833	.91283	.4242	.90557	.43994	.89803	.45554	89021	54	13
3	7	.4086	.91272	.42446	.90545	.4402	8979	4558	.89008	53	12
3	8	.40886	.9126	.42473	.90532	.44046	.89777	.45606	.88995	52	12
4	9	.40913	.91248	.42499	.9052	.44072	.89764	45632	.88981	51	12
4	10	.40939	.91236	.42525	.90507	.44098	.89752	.45658	.88968	50	12
5	11	.40966	91224	.42552	.90495	.44124	.89739	.45684	.88955	49	11
5	12	.40992	.91212	.42578	.90483	.44151	.89726	.4571	.88942	48	11
6	13	.41019	.912	.42604	.9047	.44177	.89713	.45736	.88928	47	11
6	14	.41045	.91188	.42631	.90458	.44203	.897	.45762	.88915	46	11
7	15	.41072	.91176	.42657	.90446	.44229	.89687	.45787	.88902	45	11
7	16	.41098	.91164	.42683	.90433	.44255	.89674	.45813	.88888	44	10
7	17	.41125	.91152	.42709	.90421	.44281	.89662	.45839	.88875	43	10
8	18	.41151	.9114	.42736	.90408	.44307	.89649	.45865	.88862	42	10
8	19	.41178	.91128	.42762	.90396	.44333	.89636	45891	.88848	41	10
9	20	.41204	.91116	.42788	.90383	.44359	.89623	.45917	.88835	40	9
9	21	.41231	.91104	.42815	.90371	.44385	.8961	.45942	.88822	39	9
10	22	.41257	.91092	.42841	.90358	.44411	.89597	45968	88808	38	9
10	23	.41284	.9108	.42867	.90346	.44437	.89584	.45994	.88795	37	9
10	24	.4131	91068	.42894	.90334	.44464	.89571	.4602	.88782	36	8
11	25	.41337	.91056	.4292	.90321	.4449	.89558	.46046	.88768	35	8
11	26	.41363	.91044	.42946	.90309	.44516	.89545	46072	.88755	34	8
12	27	.4139	.91032	.42972	.90296	.44542	.89532	46097	.88741	33	8
12	28	.41416	.9102	.42999	.90284	.44568	.89519	.46123	88728	32	7
13	29	.41443	.91008	.43025	90271	.44594	.89506	46149	.88715	31	7
13	30	.41469	.90996	.43051	.90259	.4462	.89493	.46175	.88701	30	7
13	31	.41496	.90984	.43077	.90246	.44646	.8948	46201	.88688	29	7
14	32	.41522	.90972	.43104	.90233	.44672	.89467	.46226	.88674	28	7
14	33	41549	9096	4313	.90221	.44698	.89454	.46252	.88661	27	6
15	34	.41575	.90948	.43156	.90208	.44724	.89441	.46278	.88647	26	6
15	35	.41602	.90936	.43182	90196	.4475	.89428	.46304	.88634	25	6
16	36	.41628	.90924	.43209	.90183	.44776	.89415	.4633	.8862	24	6
16	37	.41655	.90911	.43235	90171	44802	.89402	.46355	.88607	23	5
16	38	.41681	.90899	.43261	90158	.44828	.89389	.46381	.88593	22	5
17	39	.41707	.90887	.43287	.90146	.44854	.89376	.46407	.8858	21	5
17	40	.41734	90875	.43313	90133	.4488	.89363	.46433	.88566	20	5
18	41	.4176	.90863	.4334	.9012	.44906	.8935	.46458	.88553	19	4
18	42	.41787	.90851	43366	.90108	.44932	.89337	46484	.88539	18	4
19	43	.41813	.90839	.43392	.90095	.44958	.89324	.4651	.88526	17	4
19	44	.4184	.90826	.43418	90082	.44984	.89311	.46536	.88512	16	4
20	45	.41866	.90814	.43445	9007	.4501	.89298	.46561	.88499	15	4
20	46	.41892	.90802	.43471	.90057	.45036	.89285	.46587	.88485	14	3
20	47	.41919	.9079	.43497	.90045	.45062	.89272	.46613	.88472	13	3
21	48	.41945	.90778	.43523	.90032	.45088	.89259	.46639	.88458	12	3
21	49	.41972	.90766	.43549	.90019	.45114	.89245	.46664	.88445	11	3
22	50	.41998	.90753	.43575	.90007	4514	.89232	.4669	.88431	10	2
22	51	.42024	.90741	.43602	89994	.45166	.89219	.46716	.88417	9	2
23	52	.42051	.90729	.43628	.89981	.45192	.89206	.46742	.88404	8	2
23	53	.42077	.90717	.43654	.89968	45218	.89193	.46767	.8839	7	2
23	54	.42104	.90704	4368	.89956	.45243	.8918	.46793	.88377	6	1
24	55	.4213	.90692	43706	89943	.45269	89167	.46819	.88363	5	1
24	56	.42156	.9068	.43733	.8993	.45295	.89153	.46844	.88349	4	1
25	57	.42183	.90668	.43759	.89918	45321	.8914	.4687	.88336	3	1
25	58	.42209	.90655	.43785	.89905	.45347	.89127	.46896	.88322	2	0
26	59	.42235	.90643	.43811	.89892	.45373	.89114	46921	88308	1	0
26	60	.42262	.90631	.43837	.89879	.45399	.89101	.46947	88295	0	0
′		N. cos.	N. sine.	N. cos	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		65°		64°		63°		62°			

Prop. parts.		28°		29°		30°		31°			Prop. parts.
25		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		14
0	0	.46947	.88295	.48481	.87462	5	.86603	.51504	85717	60	14
0	1	.46973	.88281	.48506	.87448	.50025	.86588	.51529	.85702	59	14
1	2	46999	.88267	.48532	.87434	.5005	.86573	.51554	.85687	58	14
1	3	47024	.88254	.48557	.8742	.50076	.86559	.51579	.85672	57	13
2	4	.4705	.8824	.48583	.87406	.50101	.86544	.51604	.85657	56	13
2	5	.47076	88226	.48608	.87391	.50126	.8653	.51628	.85642	55	13
3	6	.47101	.88213	.48634	.87377	.50151	.86515	.51653	.85627	54	13
3	7	.47127	88199	.48659	.87363	.50176	86501	.51678	.85612	53	12
3	8	.47153	.88185	.48684	.87349	.50201	.86486	.51703	.85597	52	12
4	9	.47178	.88172	.4871	.87335	.50227	.86471	.51728	.85582	51	12
4	10	.47204	.88158	.48735	.87321	.50252	.86457	.51753	.85567	50	12
5	11	.47229	.88144	.48761	.87306	.50277	.86442	.51778	.85551	49	11
5	12	.47255	.8813	.48786	.87292	.50302	.86427	.51803	.85536	48	11
5	13	.47281	.88117	.48811	.87278	.50327	.86413	.51828	.85521	47	11
6	14	47306	.88103	.48837	.87264	.50352	.86398	.51852	.85506	46	11
6	15	.47332	.88089	.48862	.8725	.50377	.86384	.51877	.85491	45	11
7	16	.47358	.88075	.48888	.87235	.50403	.86369	.51902	.85476	44	10
7	17	.47383	.88062	.48913	.87221	.50428	.86354	.51927	.85461	43	10
8	18	.47409	.88048	.48938	.87207	.50453	.8634	.51952	.85446	42	10
8	19	.47434	.88034	.48964	.87193	.50478	.86325	.51977	.85431	41	10
8	20	.4746	.8802	.48989	.87178	.50503	.8631	.52002	.85416	40	9
9	21	.47486	.88006	.49014	.87164	.50528	.86295	.52026	.85401	39	9
9	22	.47511	87993	.4904	.8715	.50553	.86281	.52051	.85385	38	9
10	23	.47537	.87979	.49065	.87136	.50578	.86266	.52076	.8537	37	9
10	24	.47562	.87965	.4909	.87121	.50603	.86251	.52101	.85355	36	8
10	25	.47588	.87951	.49116	.87107	.50628	.86237	.52126	.8534	35	8
11	26	.47614	.87937	.49141	87093	.50654	.86222	.52151	.85325	34	8
11	27	.47639	.87923	.49166	.87079	.50679	.86207	.52175	.8531	33	8
12	28	.47665	.87909	.49192	.87064	.50704	.86192	.522	.85294	32	7
12	29	.4769	.87896	.49217	.8705	.50729	.86178	.52225	.85279	31	7
13	30	.47716	.87882	.49242	.87036	.50754	.86163	.5225	.85264	30	7
13	31	.47741	.87868	.49268	.87021	.50779	.86148	.52275	.85249	29	7
13	32	.47767	.87854	.49293	.87007	.50804	.86133	.52299	.85234	28	7
14	33	.47793	.8784	.49318	.86993	.50829	.86119	.52324	.85218	27	6
14	34	.47818	.87826	.49344	.86978	.50854	.86104	.52349	.85203	26	6
15	35	.47844	.87812	.49369	.86964	.50879	.86089	.52374	.85188	25	6
15	36	.47869	.87798	.49394	.86949	.50904	.86074	.52399	.85173	24	6
15	37	.47895	.87784	.49419	.86935	.50929	.86059	.52423	.85157	23	5
16	38	.4792	.8777	.49445	.86921	.50954	.86045	.52448	.85142	22	5
16	39	.47946	.87756	.4947	.86906	.50979	.8603	.52473	.85127	21	5
17	40	.47971	.87743	.49495	.86892	.51004	.86015	.52498	.85112	20	5
17	41	.47997	.87729	.49521	.86878	.51029	.86	.52522	.85096	19	4
18	42	.48022	.87715	.49546	.86863	.51054	.85985	.52547	.85081	18	4
18	43	.48048	.87701	.49571	.86849	.51079	.8597	.52572	.85066	17	4
18	44	.48073	.87687	.49596	.86834	.51104	.85956	.52597	.85051	16	4
19	45	.48099	.87673	.49622	.8682	.51129	.85941	.52621	.85035	15	4
19	46	.48124	.87659	.49647	.86805	.51154	.85926	.52646	8502	14	3
20	47	.4815	.87645	.49672	.86791	.51179	.85911	.52671	.85005	13	3
20	48	.48175	.87631	.49697	.86777	.51204	.85896	.52696	.84989	12	3
20	49	48201	.87617	.49723	.86762	.51229	.85881	.5272	.84974	11	3
21	50	.48226	.87603	.49748	.86748	.51254	.85866	.52745	.84959	10	2
21	51	.48252	.87589	.49773	.86733	.51279	.85851	.5277	.84943	9	2
22	52	.48277	.87575	.49798	.86719	.51304	.85836	.52794	.84928	8	2
22	53	.48303	.87561	.49824	.86704	.51329	.85821	.52819	.84913	7	2
23	54	.48328	.87546	.49849	.8669	.51354	.85806	52844	.84897	6	1
23	55	.48354	.87532	.49874	.86675	.51379	.85792	.52869	.84882	5	1
23	56	.48379	.87518	.49899	.86661	.51404	.85777	.52893	.84866	4	1
24	57	.48405	.87504	.49924	.86646	.51429	.85762	.52918	.84851	3	1
24	58	.4843	.8749	.4995	.86632	.51454	.85747	.52943	.84836	2	0
25	59	.48456	.87476	.49975	.86617	.51479	.85732	.52967	.8482	1	0
25	60	.48481	.87462	.5	.86603	.51504	.85717	.52992	.84805	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.		
		61°		60°		59°		58°			

Prop. parts	′	32°		33°		34°		35°			Prop. parts
23		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		16
0	0	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	60	16
0	1	.53017	.84789	.54488	.83851	.55943	.82887	.57381	.81899	59	16
1	2	53041	.84774	.54513	.83835	.55968	.82871	.57405	.81882	58	15
1	3	.53066	.84759	.54537	.83819	.55992	.82855	.57429	.81865	57	15
2	4	.53091	.84743	.54561	.83804	.56016	.82839	.57453	.81848	56	15
2	5	.53115	.84728	.54586	.83788	.5604	.82822	.57477	.81832	55	15
2	6	5314	.84712	.5461	.83772	.56064	.82806	.57501	.81815	54	14
3	7	.53164	.84697	.54635	.83756	.56088	.8279	.57524	.81798	53	14
3	8	.53189	.84681	.54659	.8374	.56112	.82773	.57548	.81782	52	14
3	9	.53214	.84666	.54683	.83724	.56136	.82757	.57572	.81765	51	14
4	10	.53238	.8465	.54708	.83708	.5616	.82741	.57596	.81748	50	13
4	11	.53263	.84635	.54732	.83692	.56184	.82724	.57619	.81731	49	13
5	12	.53288	.84619	.54756	.83676	.56208	.82708	57643	.81714	48	13
5	13	.53312	.84604	.54781	.8366	.56232	.82692	.57667	.81698	47	13
5	14	.53337	.84588	.54805	.83645	.56256	.82675	57691	.81681	46	12
6	15	.53361	.84573	.54829	.83629	.5628	.82659	.57715	.81664	45	12
6	16	.53386	.84557	.54854	.83613	.56305	.82643	.57738	.81647	44	12
7	17	.53411	.84542	.54878	.83597	.56329	.82626	.57762	.81631	43	11
7	18	.53435	.84526	.54902	.83581	.56353	.8261	.57786	.81614	42	11
7	19	.5346	.84511	.54927	.83565	.56377	.82593	.5781	.81597	41	11
8	20	.53484	.84495	.54951	.83549	.56401	.82577	.57833	.8158	40	11
8	21	.53509	.8448	.54975	.83533	.56425	.82561	.57857	.81563	39	10
8	22	.53534	.84464	.54999	.83517	.56449	.82544	.57881	.81546	38	10
9	23	.53558	.84448	.55024	.83501	.56473	.82528	.57904	.8153	37	10
9	24	.53583	.84433	.55048	.83485	.56497	.82511	.57928	.81513	36	10
10	25	.53607	.84417	.55072	.83469	.56521	.82495	.57952	.81496	35	9
10	26	.53632	.84402	.55097	.83453	.56545	.82478	.57976	.81479	34	9
10	27	.53656	.84386	.55121	.83437	.56569	.82462	.57999	.81462	33	9
11	28	.53681	.8437	.55145	.83421	.56593	.82446	.58023	.81445	32	9
11	29	.53705	.84355	.55169	.83405	.56617	.82429	.58047	.81428	31	8
12	30	.5373	.84339	.55194	.83389	.56641	.82413	.5807	.81412	30	8
12	31	.53754	.84324	.55218	.83373	.56665	.82396	.58094	.81395	29	8
12	32	.53779	.84308	.55242	.83356	.56689	.8238	.58118	.81378	28	7
13	33	.53804	.84292	.55266	.8334	.56713	.82363	.58141	.81361	27	7
13	34	.53828	.84277	.55291	.83324	.56736	.82347	.58165	.81344	26	7
13	35	.53853	.84261	.55315	.83308	.5676	.8233	.58189	.81327	25	7
14	36	.53877	.84245	.55339	.83292	.56784	.82314	.58212	.8131	24	6
14	37	.53902	.8423	.55363	.83276	.56808	.82297	.58236	.81293	23	6
15	38	.53926	.84214	.55388	.8326	.56832	.82281	.5826	.81276	22	6
15	39	.53951	.84198	.55412	.83244	.56856	.82264	.58283	.81259	21	6
15	40	.53975	.84182	.55436	.83228	.5688	.82248	.58307	.81242	20	5
16	41	.54	.84167	.5546	.83212	.56904	.82231	.5833	.81225	19	5
16	42	.54024	.84151	.55484	.83195	.56928	.82214	.58354	.81208	18	5
16	43	.54049	.84135	.55509	.83179	.56952	.82198	.58378	.81191	17	5
17	44	.54073	.8412	.55533	.83163	.56976	.82181	.58401	.81174	16	4
17	45	.54097	.84104	.55557	.83147	.57	.82165	.58425	.81157	15	4
18	46	.54122	.84088	.55581	.83131	.57024	.82148	.58449	.8114	14	4
18	47	.54146	.84072	.55605	.83115	.57047	.82132	.58472	.81123	13	3
18	48	.54171	.84057	.5563	.83098	.57071	.82115	.58496	.81106	12	3
19	49	.54195	.84041	.55654	.83082	.57095	.82098	.58519	.81089	11	3
19	50	.5422	.84025	.55678	.83066	.57119	.82082	.58543	.81072	10	3
20	51	.54244	.84009	.55702	.8305	.57143	.82065	.58567	.81055	9	2
20	52	.54269	.83994	.55726	.83034	.57167	.82048	.5859	.81038	8	2
20	53	.54293	.83978	.5575	.83017	.57191	.82032	.58614	.81021	7	2
21	54	.54317	83962	.55775	.83001	.57215	.82015	.58637	.81004	6	2
21	55	.54342	.83946	.55799	.82985	.57238	.81999	.58661	.80987	5	1
21	56	.54366	.8393	.55823	.82969	.57262	.81982	.58684	.8097	4	1
22	57	.54391	.83915	.55847	.82953	.57286	.81965	.58708	.80953	3	1
22	58	.54415	.83899	.55871	.82936	.5731	.81949	.58731	.80936	2	1
23	59	.5444	.83883	.55895	.8292	.57334	.81932	58755	.80919	1	0
23	60	.54464	.83867	.55919	.82904	.57358	.81915	.58779	.80902	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.		
		57°		56°		55°		54°			

Prop. parts	′	36°		37°		38°		39°			Prop. parts
23		N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.		18
0	0	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60	18
0	1	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59	18
1	2	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58	17
1	3	.58849	.8085	.60251	.79811	.61635	.78747	.63	.7766	57	17
2	4	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56	17
2	5	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55	17
2	6	.5892	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54	16
3	7	.58943	.80782	.60344	.79741	.61726	.78676	.6309	.77586	53	16
3	8	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52	16
3	9	.5899	.80748	.6039	.79706	.61772	.7864	.63135	.7755	51	15
4	10	.59014	.8073	.60414	.79688	.61795	.78622	.63158	.77531	50	15
4	11	.59037	.80713	.60437	.79671	.61818	.78604	.6318	.77513	49	15
5	12	.59061	.80696	.6046	.79653	.61841	.78586	.63203	.77494	48	14
5	13	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47	14
5	14	.59108	.80662	.60506	.79618	.61887	.7855	.63248	.77458	46	14
6	15	.59131	.80644	.60529	.796	.61909	.78532	.63271	.77439	45	14
6	16	.59154	.80627	.60553	.79583	61932	.78514	.63293	.77421	44	13
7	17	.59178	.8061	.60576	.79565	.61955	.78496	.63316	.77402	43	13
7	18	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42	13
7	19	.59225	.80576	.60622	.7953	62001	.7846	.63361	.77366	41	12
8	20	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40	12
8	21	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39	12
8	22	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.7731	38	11
9	23	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37	11
9	24	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36	11
10	25	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35	11
10	26	.59389	.80455	.60784	.79406	.6216	.78333	.63518	.77236	34	10
10	27	.59412	.80438	.60807	.79388	.62183	.78315	.6354	.77218	33	10
11	28	.59436	.8042	.6083	.79371	.62206	.78297	.63563	.77199	32	10
11	29	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31	9
12	30	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30	9
12	31	.59506	.80368	.60899	.79318	.62274	.78243	.6363	.77144	29	9
12	32	.59529	.80351	.60922	.793	.62297	.78225	.63653	.77125	28	8
13	33	.59552	.80334	.60945	.79282	.6232	.78206	.63675	.77107	27	8
13	34	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26	8
13	35	.59599	.80299	.60991	.79247	.62365	.7817	.6372	.7707	25	8
14	36	.59622	.80282	.61015	.79229	.62388	78152	.63742	.77051	24	7
14	37	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23	7
15	38	.59669	.80247	.61061	.79193	.62433	78116	63787	.77014	22	7
15	39	.59693	.8023	.61084	.79176	.62456	.78098	6381	.76996	21	6
15	40	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20	6
16	41	.59739	.80195	.6113	.7914	.62502	.78061	.63854	.76959	19	6
16	42	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.7694	18	5
16	43	.59786	.8016	.61176	.79105	.62547	.78025	.63899	.76921	17	5
17	44	.59809	.80143	.61199	.79087	.6257	.78007	.63922	.76903	16	5
17	45	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15	5
18	46	.59856	.80108	.61245	.79051	.62615	.7797	.63966	.76866	14	4
18	47	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13	4
18	48	.59902	.80073	.61291	.79016	.6266	.77934	.64011	.76828	12	4
19	49	.59926	.80056	.61314	.78998	62683	.77916	.64033	.7681	11	3
19	50	.59949	.80038	.61337	.7898	.62706	.77897	.64056	76791	10	3
20	51	.59972	.80021	.6136	.78962	.62728	.77879	.64078	.76772	9	3
20	52	59995	.80003	.61383	.78944	.62751	.77861	641	.76754	8	2
20	53	.60019	.79986	61406	.78926	.62774	.77843	.64123	.76735	7	2
21	54	.60042	.79968	.61429	.78908	.62796	.77824	64145	.76717	6	2
21	55	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5	2
21	56	.60089	.79934	.61474	.78873	62842	.77788	.6419	.76679	4	1
22	57	.60112	.79916	.61497	.78855	.62864	.77769	64212	.76661	3	1
22	58	.60135	.79899	.6152	.78837	.62887	.77751	.64234	.76642	2	1
23	59	.60158	.79881	.61543	78819	62909	.77733	.64256	.76623	1	0
23	60	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		53°		52°		51°		50°			

Prop. parts		40°		41°		42°		43°			Prop. parts
22	′	N. sine.	N. cos.	N. sine.	N. cos.	N sine.	N. cos.	N. sine.	N. cos.		19
0	0	.64279	.76604	.65606	75471	.66913	.74314	.682	.73135	60	19
0	1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	59	19
1	2	.64323	.76567	.6565	.75433	.66956	74276	.68242	.73096	58	18
1	3	.64346	.76548	65672	.75414	.66978	74256	.68264	73076	57	18
1	4	.64368	.7653	.65694	.75395	.66999	.74237	.68285	73056	56	18
2	5	.6439	.76511	.65716	.75375	.67021	.74217	.68306	73036	55	17
2	6	.64412	.76492	.65738	75356	.67043	74198	.68327	.73016	54	17
3	7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	53	17
3	8	.64457	76455	65781	.75318	.67086	.74159	.6837	.72976	52	16
3	9	.64479	76436	.65803	.75299	.67107	.74139	.68391	.72957	51	16
4	10	.64501	.76417	.65825	.7528	.67129	.7412	.68412	72937	50	16
4	11	.64524	.76398	.65847	.75261	.67151	.741	.68434	.72917	49	16
4	12	.64546	.7638	.65869	75241	.67172	.7408	.68455	.72897	48	15
5	13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	47	15
5	14	.6459	.76342	.65913	.75203	.67215	.74041	.68497	72857	46	15
6	15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	45	14
6	16	.64635	.76304	65956	.75165	.67258	.74002	.68539	.72817	44	14
6	17	.64657	.76286	.65978	.75146	.6728	.73983	.68561	.72797	43	14
7	18	.64679	.76267	.66	.75126	.67301	.73963	.68582	72777	42	13
7	19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	41	13
7	20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	72737	40	13
8	21	.64746	.7621	.66066	.75069	.67366	.73904	.68645	72717	39	12
8	22	.64768	.76192	.66088	.7505	.67387	.73885	.68666	.72697	38	12
8	23	.6479	.76173	.66109	.7503	.67409	.73865	.68688	72677	37	12
9	24	.64812	.76154	.66131	.75011	.6743	73846	.68709	.72657	36	11
9	25	.64834	.76135	.66153	.74992	.67452	.73826	.6873	72637	35	1.
10	26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	72617	34	11
10	27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	33	10
10	28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	72577	32	10
11	29	.64923	.76059	.6624	.74915	.67538	.73747	.68814	.72557	31	10
11	30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	30	10
11	31	.64967	.76022	.66284	.74876	.6758	.73708	.68857	.72517	29	9
12	32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	28	9
12	33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	27	9
12	34	.65033	.75965	.66349	.74818	.67645	.73649	.6892	.72457	26	8
13	35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	72437	25	8
13	36	.65077	.75927	.66393	.7478	.67688	.7361	.68962	.72417	24	8
14	37	.651	.75908	6414	.7476	.67709	.7359	.68983	72397	23	7
14	38	.65122	.75889	66436	74741	.6773	.7357	.69004	.72377	22	7
14	39	.65144	.7587	.66458	.74722	.67752	.73551	.69025	.72357	21	7
15	40	.65166	.75851	.6648	.74703	.67773	.73531	.69046	.72337	20	6
15	41	.65188	75832	.66501	.74683	.67795	.73511	.69067	.72317	19	6
15	42	.6521	.75813	.66523	.74664	.67816	.73491	.69088	.72297	18	6
16	43	.65232	.75794	.66545	74644	.67837	.73472	.69109	.72277	17	5
16	44	.65254	.75775	.66566	.74625	.67859	.73452	.6913	.72257	16	5
17	45	.65276	.75756	66588	.74606	.6788	.73432	.69151	.72236	15	5
17	46	.65298	.75738	.6661	.74586	.67901	.73413	.69172	.72216	14	4
17	47	.6532	.75719	.66632	.74567	.67923	.73393	.69193	.72196	13	4
18	48	.65342	.757	.66653	.74548	.67944	.73373	.69214	.72176	12	4
18	49	.65364	7568	.66675	.74528	.67965	.73353	.69235	72156	11	3
18	50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	10	3
19	51	.65408	.75642	.66718	.74489	.68008	.73314	69277	72116	9	3
19	52	.6543	.75623	.6674	7447	.68029	.73294	.69298	72095	8	3
19	53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	72075	7	2
20	54	.65474	.75585	.66783	.74431	.68072	.73254	.6934	.72055	6	2
20	55	.65496	.75566	.66805	.74412	.68093	.7323	.69361	.72035	5	2
21	56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	4	1
21	57	.6554	.75528	66848	.74373	.68136	.73195	69403	.71995	3	1
21	58	.65562	.75509	.6687	74353	68157	.73175	.69424	.71974	2	1
22	59	.65584	.7549	.66891	74334	68179	.73155	69445	.71954	1	0
22	60	.65606	.75471	.66913	.74314	.682	.73135	69466	.71934	0	0
		N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	N. cos.	N. sine.	′	
		49°		48°		47°		46°			

Prop. parts		44°			Prop. parts	Prop. parts		44°			Prop. parts
11		N. sine.	N. cos.		19	22		N. sine.	N. cos.		9
0	0	.69466	.71934	60	19	11	31	.70112	.71305	29	9
0	1	.69487	.71914	59	19	12	32	.70132	.71284	28	9
1	2	.69508	.71894	58	18	12	33	.70153	.71264	27	9
1	3	.69529	.71873	57	18	12	34	.70174	.71243	26	8
1	4	.69549	.71853	56	18	13	35	.70195	.71223	25	8
2	5	.6957	.71833	55	17	13	36	.70215	.71203	24	8
2	6	.69591	.71813	54	17	14	37	.70236	.71182	23	7
3	7	.69612	.71792	53	17	14	38	.70257	.71162	22	7
3	8	.69633	.71772	52	16	14	39	.70277	.71141	21	7
3	9	.69654	.71752	51	16	15	40	.70298	.71121	20	6
4	10	.69675	.71732	50	16	15	41	.70319	.711	19	6
4	11	.69696	.71711	49	16	15	42	.70339	.7108	18	6
4	12	.69717	.71691	48	15	16	43	.7036	.71059	17	5
5	13	.69737	.71671	47	15	16	44	.70381	.71039	16	5
5	14	.69758	.7165	46	15	17	45	.70401	.71019	15	5
6	15	.69779	.7163	45	14	17	46	.70422	.70998	14	4
6	16	.698	.7161	44	14	17	47	.70443	.70978	13	4
6	17	.69821	.7159	43	14	18	48	.70463	.70957	12	4
7	18	.69842	.71569	42	13	18	49	.70484	.70937	11	3
7	19	.69862	.71549	41	13	18	50	.70505	.70916	10	3
7	20	.69883	.71529	40	13	19	51	.70525	.70896	9	3
8	21	.69904	.71508	39	12	19	52	.70546	.70875	8	3
8	22	.69925	.71488	38	12	19	53	.70567	.70855	7	2
8	23	.69946	.71468	37	12	20	54	.70587	.70834	6	2
9	24	.69966	.71447	36	11	20	55	.70608	.70813	5	2
9	25	.69987	.71427	35	11	21	56	.70628	.70793	4	1
10	26	.70008	.71407	34	11	21	57	.70649	.70772	3	1
10	27	.70029	.71386	33	10	21	58	7067	.70752	2	1
10	28	.70049	.71366	32	10	22	59	.7069	.70731	1	0
11	29	.7007	.71345	31	10	22	60	.70711	.70711	0	0
11	30	.70091	.71325	30	10						
		N. cos.	N. sine.					N. cos.	N. sine.		
		45°						45°			

NATURAL TANGENTS AND COTANGENTS.

	0°		1°		2°		3°		
′	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	′
0	.00000	Infinite.	.01746	57.29	03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	6.3506	.03521	8.3994	.0527	8.9755	59
2	.00058	1718.87	.01804	5.4415	.0355	8.1664	.05299	8.8711	58
3	.00087	145.92	.01833	4.5613	.03579	7.9372	.05328	8.7678	57
4	.00116	859.436	.01862	3.7086	03609	7.7117	.05357	8.6656	56
5	.00145	687.549	.01891	52.8821	03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.0192	2.0807	.03667	7.2715	.05416	8.4645	54
7	.00204	491.106	.01949	1.3032	.03696	7.0566	.05445	8.3655	53
8	.00233	29.718	.01978	0.5485	03725	6.845	.05474	8.2677	52
9	.00262	381.971	.02007	49.8157	.03754	6.6367	.05503	8.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.075	50
11	.0032	12.521	.02066	8.4121	.03812	6.2296	.05562	7.9802	49
12	.00349	286.478	.02095	7.7395	.03842	6.0307	.05591	7.8863	48
13	.00378	64.441	.02124	7.0853	.03871	5.8348	.0562	7.7934	47
14	.00407	45.552	.02153	6.4489	.039	5.6418	.05649	7.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	14.858	.02211	5.2261	.03958	5.2644	.05708	7.5205	44
17	.00495	02.219	.0224	4.6386	.03987	5.0798	.057·37	7.4314	43
18	.00524	190.984	.02269	4.0661	.04016	4.8978	.05766	7.3432	42
19	.00553	80.932	.02298	3.5081	.04046	4.7185	.05795	7.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	63.7	.02357	2.4335	.04104	4.3075	.05854	7.0837	39
22	.0064	56.259	.02386	1.9158	.04133	4.1957	.05883	6.999	38
23	00669	49.465	.02415	1.4106	.04162	4.0263	.05912	6.915	37
24	.00698	43.237	.02444	0.9174	04191	3.8593	.05941	6.8319	36
25	.00727	137.507	.02473	40.4358	0422	23.6945	.0597	16.7496	35
26	.00756	32.219	.02502	39.9655	0425	3.5321	.05999	6.6681	34
27	.00785	27.321	.02531	9.5059	.04279	3.3718	.06029	6.5874	33
28	.00814	22.774	.0256	9.0568	.04308	3.2137	.06058	6.5075	32
29	.00844	18.54	.02589	8.6177	.04337	3.0577	.06087	6.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	10.892	.02648	7.7686	.04395	2.7519	.06145	6.2722	29
32	.00931	07.426	.02677	7.3579	.04424	2.602	.06175	6.1952	28
33	.0096	04.171	.02706	6.956	.04454	2.4541	.06204	6.119	27
34	00989	01.107	.02735	6.5627	.04483	2.3081	.06233	6.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.164	.06262	15.9687	25
36	.01047	5.4895	.02793	5.8006	.04541	2.0217	.06291	5.8945	24
37	.01076	2.9085	.02822	5.4313	.0457	1.8813	.06321	5.8211	23
38	.01105	0.4633	.02851	5.0695	.04599	1.7426	.0635	5.7483	22
39	.01135	88.1436	.02881	4.7151	.04628	1.6056	.06379	5.6762	21
40	.01164	85.9398	.0291	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	3.8435	.02939	4.0273	.04687	1.3369	.06437	5.534	19
42	.01222	1.847	.02968	3.6935	.04716	1.2049	.06467	5.4638	18
43	.01251	79.9434	.02997	3.3662	.04745	1.0747	.06496	5.3943	17
44	.0128	8.1263	.03026	3.0452	.04774	0.946	.06525	5.3254	16
45	.01309	76.39	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	4.7292	.03084	2.4213	.04832	0.6932	.06584	5.1893	14
47	.01367	3.139	.03114	2,1181	.04862	0.5691	.06613	5.1222	13
48	.01396	1.6151	03143	1.8205	.04891	0.4465	.06642	5.0557	12
49	.01425	0.1533	.03172	1.5284	.0492	0.3253	.06671	4.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.067	14.9244	10
51	.01484	7.4019	.0323	0.9599	.04978	0.0872	.0673	4.8596	9
52	.01513	6.1055	.03259	0.6833	.05007	19.9702	.06759	4.7954	8
53	.01542	4.858	.03288	0.4116	.05037	9.8546	.06788	4.7317	7
54	.01571	3.6567	03317	0.1446	.05066	9.7403	.06817	4.6685	6
55	.016	62.4992	.03346	29.8823	05095	19.6273	.06847	14.6059	5
56	.01629	1.3829	.03376	9.6245	.05124	9.5156	.06876	4.5438	4
57	.01658	0.3058	.03405	9.3711	.05153	9.4051	.06905	4.4823	3
58	.01687	59.2659	.03434	9.122	.05182	9.2959	.06934	4.4212	2
59	.01716	8.2612	.03463	8.8771	.05212	9.1879	.06963	4.3607	1
60	.01746	57.29	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	89°		88°		87°		86°		

′	4°		5°		6°		7°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.06993	14.3007	.08749	11.4301	.1051	9.51436	.12278	8.14435	60
1	.07022	4.2411	.08778	1.3919	.1054	.48781	.12308	.12481	59
2	.07051	4.1821	.08807	1.354	.10569	.46141	.12338	.10536	58
3	.0708	4.1235	.08837	1.3163	.10599	.43515	.12367	.086	57
4	.0711	4.0655	.08866	1.2789	.10628	.40904	.12397	.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	3.9507	.08925	1.2048	.10687	.35724	.12456	.02848	54
7	.07197	3.894	.08954	1.1681	.10716	.33154	.12485	8.00948	53
8	.07227	3.8378	.08983	1.1316	.10746	.30599	.12515	7.99058	52
9	.07256	3.7821	.09013	1.0954	.10775	.28058	.12544	.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.2553	12574	7.95302	50
11	.07314	3.6719	.09071	1.0237	.10834	.23016	.12603	.93438	49
12	.07344	3.6174	.09101	0.9882	.10863	.20516	12633	.91582	48
13	.07373	3.5634	.0913	0.9529	.10893	.18028	12662	.89734	47
14	.07402	3.5098	.09159	0.9178	.10922	.15554	.12692	.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	3.4039	.09218	0.8483	.10981	.10646	.12751	.84242	44
17	.0749	3.3515	.09247	0.8139	.11011	.08211	.12781	82428	43
18	.07519	3.2996	.09277	0.7797	.1104	.05789	.1281	80622	42
19	.07548	3.248	.09306	0.7457	.1107	.03379	.1284	78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7 77035	40
21	.07607	3.1461	.09365	0.6783	.11128	8.98598	.12899	.75254	39
22	.07636	3.0958	09394	0.645	.11158	.96227	.12929	7348	38
23	.07665	3.0458	.09423	0.6118	.11187	.93867	.12958	71715	37
24	.07695	2.9967	.09453	0.5789	.11217	.9152	.12988	.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	13017	7 68208	35
26	.07753	2.8981	.09511	0.5136	.11276	.86862	.13047	66466	34
27	.07782	2.8496	.09541	0.4813	.11305	.84551	.13076	64732	33
28	.07812	2.8014	.0957	0.4491	.11335	.82252	13106	.63005	32
29	.07841	2.7536	.096	0.4172	.11364	.79964	.13136	.61287	31
30	.0787	12.7062	.09629	10.3854	.11394	8.77689	13165	7 59575	30
31	.07899	2.6591	.09658	0.3538	.11423	75425	13195	57872	29
32	.07929	2.6124	.09688	0.3224	.11452	.73172	13224	56176	28
33	.07958	2.566	.09717	0.2913	.11482	.70931	.13254	54487	27
34	.07987	2.5199	.09746	0.2602	.11511	.68701	.13284	52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	2.4288	.09805	0.1988	.1157	.64275	.13343	.49465	24
37	.08075	2.3838	.09834	0.1683	.116	.62078	.13372	.47806	23
38	.08104	2.339	.09864	0.1381	.11629	.59893	.13402	.46154	22
39	.08134	2.2946	.09893	0.108	.11659	.57718	.13432	.44509	21
40	.08163	12.2505	.09923	10.078	.11688	8.55555	13461	7 42871	20
41	.08192	2.2067	.09952	0.0483	.11718	.53402	.13491	.4124	19
42	.08221	2.1632	.09981	0.0187	.11747	.51259	.13521	.39616	18
43	.08251	2.1201	.10011	9.9893	.11777	.49128	.1355	37999	17
44	.0828	2.0772	.1004	.96007	.11806	.47007	.1358	.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	1.9923	.10099	.90211	.11865	.42795	.13639	.3319	14
47	08368	1.9504	.10128	.87338	.11895	.40705	.13669	.316	13
48	.08397	1.9087	.10158	.84482	.11924	.38625	.13698	.30018	12
49	.08427	1 8673	.10187	.81641	.11954	.36555	.13728	.28442	11
50	.08456	11.8262	10216	9.78817	11983	8.34496	.13758	7.26873	10
51	.08485	1.7853	10246	.76009	.12013	.32446	13787	.2531	9
52	.08514	1.7448	10275	.73217	.12042	.30406	.13817	.23754	8
53	.08544	1.7045	10305	.70441	.12072	.28376	.13846	.22204	7
54	.08573	1.6645	10334	.6768	.12101	.26355	.13876	.20661	6
55	.08602	11.6248	10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	1.5853	10393	.62205	.1216	.22344	.13935	.17594	4
57	.08661	1.5461	.10422	.5949	.1219	.20352	13965	.16071	3
58	.0869	1.5072	.10452	.56791	.12219	.1837	.13995	.14553	2
59	.0872	1.4685	10481	.54106	.12249	.16398	.14024	.13042	1
60	.08749	11.4301	1051	9.51436	.12278	8.14435	.14054	7.11537	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	85°		84°		83°		82°		

	8°		9°		10°		11°		
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.140 54	7.115 37	.158 38	6.313 75	.176 33	5.671 28	.194 38	5.144 55	60
1	.140 84	.100 38	.158 68	.301 89	.176 63	.661 65	.194 68	136 58	59
2	.141 13	.085 46	.158 98	.290 07	.176 93	.652 05	194 98	128 62	58
3	.141 43	.070 59	.159 28	.278 29	.177 23	.642 48	.195 29	.120 69	57
4	.141 73	.055 79	.159 58	.266 55	.177 53	.632 95	.195 59	.112 79	56
5	.142 02	7.041 05	.159 88	6.254 86	.177 83	5.623 44	.195 89	5.104 9	55
6	.142 32	.026 37	.160 17	.243 21	.178 13	.613 97	.196 19	.097 04	54
7	.142 62	.011 74	.160 47	.231 6	.178 43	.604 52	.196 49	.089 21	53
8	.142 91	6.997 18	.160 77	.220 03	.178 73	.595 11	.196 8	.081 39	52
9	.143 21	.982 68	.161 07	.208 51	.179 03	.585 73	.197 1	.073 6	51
10	143 51	6.968 23	.161 37	6.197 03	.179 33	5.576 38	.197 4	5.065 84	50
11	.143 81	.953 85	.161 67	.185 59	.179 63	.567 06	.197 7	.058 09	49
12	.144 1	.939 52	.161 96	.174 19	.179 93	.557 77	.198 01	.050 37	48
13	.144 4	.925 25	.162 26	.162 83	.180 23	.548 51	.198 31	.042 67	47
14	144 7	.911 04	.162 56	.151 51	.180 53	.539 27	.198 61	.034 99	46
15	.144 99	6.896 88	.162 86	6.140 23	.180 83	5.530 07	.198 91	5.027 34	45
16	.145 29	.882 78	.163 16	.128 99	181 13	.520 9	.199 21	.019 71	44
17	.145 59	.868 74	.163 46	.117 79	.181 43	.511 76	.199 52	.012 1	43
18	.145 88	.854 75	.163 76	.106 64	.181 73	.502 64	.199 82	.004 51	42
19	.146 18	.840 82	.164 05	.095 52	.182 03	.493 56	.200 12	4.996 95	41
20	.146 48	6.826 94	.164 35	6.084 44	.182 33	5.484 51	.200 42	4.989 4	40
21	.146 78	.813 12	.164 65	.073 4	.182 63	.475 48	.200 73	.981 88	39
22	.147 07	.799 36	.164 95	.062 4	.182 93	466 48	.201 03	974 38	38
23	.147 37	.785 64	.165 25	.051 43	.183 23	.457 51	.201 33	.966 9	37
24	.147 67	.771 99	.165 55	.040 51	.183 53	.448 57	.201 64	.959 45	36
25	.147 96	6.758 38	.165 85	6.029 62	.183 83	5.439 66	.201 94	4.952 01	35
26	.148 26	.744 83	.166 15	.018 78	.184 14	.430 77	.202 24	.944 6	34
27	.148 56	.731 33	.166 45	.007 97	.184 44	.421 92	.202 54	.937 21	33
28	.148 86	.717 89	.166 74	5.997 2	.184 74	413 09	.202 85	.929 84	32
29	.149 15	.704 5	.167 04	.986 46	185 04	.404 29	.203 15	.922 49	31
30	.149 45	6.691 16	.167 34	5.975 76	.185 34	5.395 52	.203 45	4.915 16	30
31	.149 75	.677 87	.167 64	.965 1	.185 64	.386 77	.203 76	.907 85	29
32	.150 05	.664 63	.167 94	.954 48	.185 94	.378 05	.204 06	.900 56	28
33	.150 34	.651 44	.168 24	.943 9	.186 24	.369 36	.204 36	.893 3	27
34	.150 64	.638 31	.168 54	.933 35	.186 54	.360 7	.204 66	.886 05	26
35	.150 94	6.625 23	.168 84	5.922 83	.186 84	5.352 06	.204 97	4.878 82	25
36	.151 24	.612 19	.169 14	.912 35	.187 14	.343 45	.205 27	.871 62	24
37	.151 53	.599 21	.169 44	.901 91	.187 45	.334 87	.205 57	.864 44	23
38	.151 83	.586 27	.169 74	.891 51	.187 75	.326 31	.205 88	857 27	22
39	.152 13	.573 39	.170 04	.881 14	.188 05	.317 78	.206 18	850 13	21
40	.152 43	6.560 55	.170 33	5.870 8	.188 35	5.309 28	.206 48	4.843	20
41	.152 72	.547 77	.170 63	.860 51	.188 65	.300 8	.206 79	.835 9	19
42	.153 02	.535 03	.170 93	.850 24	.188 95	.292 35	.207 09	.828 82	18
43	.153 32	.522 34	.171 23	.840 01	.189 25	.283 93	.207 39	.821 75	17
44	153 62	.509 7	.171 53	.829 82	.189 55	.275 53	.207 7	.814 71	16
45	.153 91	6.497 1	.171 83	5.819 66	.189 86	5.267 15	.208	4.807 69	15
46	154 21	.484 56	.172 13	.809 53	.190 16	.258 8	.208 3	.800 68	14
47	.154 51	.472 06	.172 43	.799 44	.190 46	.250 48	.208 61	.793 7	13
48	.154 81	459 61	.172 73	.789 38	190 76	.242 18	.208 91	.786 73	12
49	.155 11	.447 2	.173 03	.779 36	.191 06	.233 91	.209 21	.779 78	11
50	.155 4	6.434 84	.173 33	5.769 37	.191 36	5.225 66	.209 52	4.772 86	10
51	.155 7	.422 53	.173 63	.759 41	.191 66	.217 44	.209 82	.765 95	9
52	156	410 26	.173 93	.749 49	.191 97	.209 25	.210 13	.759 06	8
53	156 3	.398 04	.174 23	.739 6	.192 27	.201 07	.210 43	.752 19	7
54	156 6	.385 87	.174 53	.729 74	.192 57	.192 93	.210 73	.745 34	6
55	.156 89	6.373 74	.174 83	5.719 92	.192 87	5.184 8	.211 04	4.738 51	5
56	.157 19	.361 65	.175 13	.710 13	.193 17	.176 71	.211 34	.731 7	4
57	.157 49	.349 61	.175 43	.700 37	.193 47	.168 63	.211 64	.724 9	3
58	157 79	.337 61	175 73	.690 64	.193 78	.160 58	.211 95	.718 13	2
59	.158 09	.325 66	.176 03	.680 94	.194 08	.152 56	212 25	.711 37	1
60	.158 38	6.313 75	.176 33	5.671 28	.194 38	5.144 55	.212 56	4.704 63	0
	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	
	81°		80°		79°		78°		

′	12°		13°		14°		15°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	.69791	.23117	.32573	.24964	.00582	.26826	.72771	59
2	.21316	.69121	.23148	.32001	.24995	.00086	.26857	.72338	58
3	.21347	.68452	.23179	.3143	.25026	3.99592	.26888	.71907	57
4	.21377	.67786	.23209	.3086	.25056	.99099	.2692	.71476	56
5	.21408	4.67121	.2324	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	.66458	.23271	.29724	.25118	.98117	.26982	.70616	54
7	.21469	.65797	.23301	.29159	.25149	.97627	.27013	.70188	53
8	.21499	.65138	.23332	.28595	.2518	.97139	.27044	.69761	52
9	.21529	.6448	.23363	.28032	.25211	.96651	.27076	.69335	51
10	.2156	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.2159	.63171	.23424	.26911	.25273	.9568	.27138	.68485	49
12	.21621	.62518	.23455	.26352	.25304	.95196	.27169	.68061	48
13	.21651	.61868	.23485	.25795	.25335	.94713	.27201	.67638	47
14	.21682	.61219	.23516	.25239	.25366	.94232	.27232	.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	21743	.59927	.23578	.24132	.25428	.93271	.27294	.66376	44
17	.21773	.59283	23608	.2358	.25459	.92793	.27326	.65957	43
18	.21804	.58641	23639	.2303	.2549	.92316	.27357	.65538	42
19	.21834	.58001	2367	.22481	.25521	.91839	.27388	.65121	41
20	.21864	4.57363	.237	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	.56726	.23731	.21387	.25583	.9089	.27451	.64289	39
22	.21925	.56091	.23762	.20842	.25614	.90417	.27482	.63874	38
23	.21956	.55458	.23793	.20298	.25645	.89945	.27513	.63461	37
24	.21986	.54826	.23823	.19756	.25676	.89474	.27545	.63048	36
25	.22017	4.54196	23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	.53568	.23885	.18675	.25738	.88536	.27607	.62224	34
27	.22078	.52941	.23916	.18137	.25769	.88068	.27638	.61814	33
28	22108	.52316	.23946	.176	.258	.87601	.2767	.61405	32
29	.22139	.51693	23977	.17064	.25831	.87136	.27701	.60996	31
30	.22169	4.51071	.24008	4.1653	.25862	3.86671	.27732	3.60588	30
31	222	.50451	.24039	.15997	.25893	.86208	.27764	.60181	29
32	.22231	.49832	24069	.15465	.25924	.85745	.27795	.59775	28
33	22261	.49215	.241	.14934	.25955	.85284	.27826	.5937	27
34	.22292	.486	.24131	.14405	.25986	.84824	.27858	.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	.47374	.24193	1335	.26048	.83906	2792	.5816	24
37	.22383	.46764	.24223	.12825	.26079	.83449	.27952	.57758	23
38	.22414	.46155	.24254	.12301	.2611	.82992	.27983	.57357	22
39	.22444	.45548	.24285	.11778	.26141	.82537	.28015	.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	.44338	.24347	.10736	.26203	.8163	.28077	.56159	19
42	.22536	.43735	.24377	.10216	.26235	.81177	.28109	.55761	18
43	.22567	.43134	.24408	.09699	.26266	.80726	.2814	.55364	17
44	.22597	42534	.24439	.09182	.26297	.80276	.28172	.54968	16
45	.22628	4.41936	.2447	4.08666	.26328	3.79827	28203	3.54573	15
46	.22658	.4134	.24501	.08152	.26359	.79378	.28234	.54179	14
47	22689	.40745	.24532	.07639	.2639	78931	.28266	.53785	13
48	22719	.40152	.24562	.07127	.26421	.78485	28297	.53393	12
49	2275	3956	.24593	.06616	.26452	.7804	.28329	.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.2836	3.52609	10
51	.22811	38381	.24655	.05599	.26515	.77152	.28391	.52219	9
52	.22842	.37793	.24686	.05092	.26546	76709	.28423	.51829	8
53	22872	.37207	.24717	.04586	.26577	.76268	28454	.51441	7
54	22903	.36623	.24747	.04081	26608	.75828	.28486	.51053	6
55	.22934	4.3604	24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	22964	.35459	.24809	.03075	.2667	.7495	.28549	.50279	4
57	22995	34879	.2484	.02574	26701	.74512	2858	.49894	3
58	23026	343	24871	.02074	.26733	.74075	.28612	.49509	2
59	23056	33723	.24902	.01576	.26764	.7364	.28643	.49125	1
60	.23087	4.33148	.24933	4.01078	26795	3.73205	.28675	3.48741	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	77°		76°		75°		74°		

′	16°		17°		18°		19°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	.48359	.30605	.26745	.32524	.07464	.34465	.90147	59
2	.28738	.47977	.30637	.26406	.32556	.0716	.34498	.89873	58
3	.28769	.47596	.30669	26067	.32588	.06857	.3453	.896	57
4	.288	.47216	.307	.25729	.32621	.06554	.34563	.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	.46458	.30764	.25055	.32685	.0595	.34628	.88783	54
7	.28895	.4608	.30796	.24719	.32717	.05649	.34661	.88511	53
8	.28927	.45703	.30828	.24383	.32749	.05349	.34693	.8824	52
9	.28958	.45327	.3086	.24049	.32782	.05049	.34726	.8797	51
10	.2899	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.877.	50
11	.29021	.44576	.30923	.23381	.32846	.0445	.34791	.8743	49
12	.29053	.44202	.30955	.23048	.32878	.04152	.34824	.87161	48
13	.29084	.43829	.30987	.22715	.32911	.03854	.34856	.86892	47
14	.29116	.43456	.31019	.22384	.32943	.03556	.34889	.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.0326	.34922	2.86356	45
16	.29179	.42713	.31083	.21722	.33007	.02963	.34954	.86089	44
17	.2921	.42343	.31115	.21392	.3304	.02667	.34987	.85822	43
18	.29242	.41973	.31147	.21063	.33072	.02372	.35019	.85555	42
19	.29274	.41604	.31178	.20734	.33104	.02077	.35052	.85289	41
20	.29305	3.41236	.3121	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	.40869	.31242	.20079	.33169	.01489	.35117	.84758	39
22	.29368	.40502	.31274	.19752	.33201	.01196	.3515	.84494	38
23	.294	.40136	.31306	.19426	.33233	.00903	.35183	.84229	37
24	.29432	.39771	.31338	.191	.33266	.00611	.35216	.83965	36
25	.29463	3.39406	.3137	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	.39042	.31402	.18451	.3333	.00028	.35281	.83439	34
27	.29526	.38679	.31434	.18127	.33363	2.99738	.35314	.83176	33
28	.29558	.38317	.31466	.17804	.33395	.99447	.35346	.82914	32
29	.2959	.37955	.31498	.17481	.33427	.99158	.35379	.82653	31
30	.29621	3.37594	.3153	3.17159	.3346	2.98868	.35412	2.82391	30
31	.29653	.37234	.31562	.16838	.33492	.9858	.35445	.8213	29
32	.29685	.36875	.31594	.16517	.33524	.98292	.35477	.8187	28
33	.29716	.36516	.31626	.16197	.33557	.98004	.3551	.8161	27
34	.29748	.36158	.31658	.15877	.33589	.97717	.35543	.8135	26
35	.2978	3.358	.3169	3.15558	.33621	2.9743	.35576	2.81091	25
36	.29811	.35443	.31722	.1524	.33654	.97144	.35608	.80833	24
37	.29843	.35087	.31754	.14922	.33686	.96858	.35641	.80574	23
38	.29875	.34732	.31786	.14605	.33718	.96573	.35674	.80316	22
39	.29906	.34377	.31818	.14288	.33751	.96288	.35707	.80059	21
40	.29938	3.34023	.3185	3.13972	.33783	2.96004	.3574	2.79802	20
41	.2997	.3367	.31882	.13656	.33816	.95721	.35772	.79545	19
42	.30001	.33317	.31914	.13341	.33848	.95437	.35805	.79289	18
43	.30033	.32965	.31946	.13027	.33881	.95155	.35838	.79033	17
44	.30065	.32614	.31978	.12713	.33913	.94872	.35871	.78778	16
45	.30097	3.32264	.3201	3.124	.33945	2.9459	.35904	2.78523	15
46	.30128	.31914	.32042	.12087	.33978	.94309	.35937	.78269	14
47	.3016	.31565	.32074	.11775	.3401	.94028	.35969	.78014	13
48	.30192	.31216	.32106	.11464	.34043	.93748	.36002	.77761	12
49	.30224	.30868	.32139	.11153	.34075	.93468	.36035	.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	.30174	.32203	.10532	.3414	.9291	.36101	.77002	9
52	.30319	.29829	.32235	.10223	.34173	.92632	.36134	.7675	8
53	.30351	.29483	.32267	.09914	.34205	.92354	.36167	.76498	7
54	.30382	.29139	.32299	.09606	.34238	.92076	.36199	.76247	6
55	.30414	3.28795	.32331	3.09298	.3427	2.91799	.36232	2.75996	5
56	.30446	.28452	.32363	.08991	.34303	.91523	.36265	.75746	4
57	.30478	.28109	.32396	.08685	.34335	.91246	.36298	.75496	3
58	.30509	.27767	.32428	.08379	.34368	.90971	.36331	.75246	2
59	.30541	.27426	.3246	.08073	.344	.90696	.36364	.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	73°		72°		71°		70°		

	20°		21°		22°		23°		
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.3643	.74499	.3842	.60283	.40436	.47302	.42482	.35395	59
2	.36463	.74251	.38453	.60057	.4047	.47095	.42516	.35205	58
3	.36496	.74004	.38487	.59831	.40504	.46888	.42551	.35015	57
4	36529	.73756	.3852	.59606	.40538	.46682	.42585	.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	.73263	.38587	.59156	.40606	.4627	.42654	.34447	54
7	.36628	.73017	.3862	.58932	.4064	.46065	.42688	.34258	53
8	.36661	.72771	.38654	.58708	.40674	.4586	.42722	.34069	52
9	.36694	.72526	.38687	.58484	.40707	.45655	.42757	.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.3676	.72036	.38754	.58038	.40775	.45246	.42826	.33505	49
12	.36793	.71792	.38787	.57815	.40809	.45043	.4286	.33317	48
13	.36826	.71548	.38821	.57593	.40843	.44839	.42894	.3313	47
14	.36859	.71305	.38854	.57371	.40877	.44636	.42929	.32943	46
15	.36892	2.71062	.38888	2.5715	.40911	2.44433	.42963	2.32756	45
16	.36925	.70819	.38921	.56928	.40945	.4423	.42998	.3257	44
17	.36958	.70577	.38955	.56707	.40979	.44027	.43032	.32383	43
18	.36991	.70335	.38988	.56487	.41013	.43825	.43067	.32197	42
19	.37024	.70094	.39022	.56266	.41047	.43623	.43101	.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.3709	.69612	.39089	.55827	.41115	.4322	.4317	.31641	39
22	.37124	.69371	.39122	.55608	.41149	.43019	.43205	.31456	38
23	.37157	.69131	.39156	.55389	.41183	.42819	.43239	.31271	37
24	.3719	.68892	.3919	.5517	.41217	.42618	.43274	.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	.68414	.39257	.54734	.41285	.42218	.43343	.30718	34
27	.37289	.68175	.3929	.54516	.41319	.42019	43378	.30534	33
28	.37322	.67937	.39324	.54299	.41353	.41819	.43412	.30351	32
29	.37355	.677	.39357	.54082	.41387	.4162	.43447	.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	.67225	.39425	.53648	.41455	.41223	.43516	.29801	29
32	.37455	.66989	.39458	.53432	.4149	.41025	.4355	.29619	28
33	.37488	.66752	.39492	.53217	.41524	.40827	.43585	.29437	27
34	.37521	.66516	.39526	.53001	.41558	.40629	.4362	.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	.66046	.39593	.52571	.41626	.40235	.43689	.28891	24
37	.37621	.65811	.39626	.52357	.4166	.40038	.43724	.2871	23
38	.37654	.65576	.3966	.52142	.41694	.39841	.43758	.28528	22
39	.37687	.65342	.39694	.51929	41728	.39645	.43793	.28348	21
40	.3772	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	.64875	.39761	.51502	.41797	.39253	.43862	.27987	19
42	.37787	.64642	.39795	.51289	.41831	.39058	.43897	.27806	18
43	.3782	.6441	.39829	.51076	.41865	.38862	.43932	.27626	17
44	.37853	.64177	.39862	.50864	.41899	.38668	.43966	27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.3792	.63714	.3993	.5044	.41968	.38279	.44036	.27088	14
47	.37953	.63483	.39963	.50229	.42002	.38084	.44071	.26909	13
48	.37986	.63252	.39997	.50018	.42036	.37891	.44105	.2673	12
49	.3802	.63021	.40031	.49807	.4207	.37697	.4414	.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	.62561	.40098	.49386	.42139	.37311	.4421	.26196	9
52	.3812	.62332	.40132	.49177	.42173	.37118	.44244	.26018	8
53	.38153	.62103	.40166	.48967	.42207	.36925	.44279	.2584	7
54	.38186	.61874	.402	.48758	.42242	.36733	.44314	.25663	6
55	.3822	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	.61418	.40267	.4834	.4231	.36349	.44384	.25309	4
57	.38286	.6119	.40301	.48132	.42345	.36158	.44418	.25132	3
58	.3832	60963	.40335	.47924	.42379	.35967	.44453	.24956	2
59	.38353	.60736	.40369	.47716	.42413	.35776	.44488	.2478	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	69°		68°		67°		66°		

′	24°		25°		26°		27°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.44523	2.24604	.46631	2.14451	.48773	2.0503	.50953	1.96261	60
1	.44558	.24428	.46666	.14288	.48809	.04879	.50989	.9612	59
2	.44593	.24252	.46702	.14125	.48845	.04728	.51026	.95979	58
3	.44627	.24077	.46737	.13963	.48881	.04577	.51063	.95838	57
4	.44662	.23902	.46772	.13801	.48917	.04426	.51099	.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	.23553	.46843	.13477	.48989	.04125	.51173	.95417	54
7	.44767	.23378	.46879	.13316	.49026	.03975	.51209	.95277	53
8	.44802	.23204	.46914	.13154	.49062	.03825	.51246	.95137	52
9	.44837	.2303	.4695	.12993	.49098	.03675	.51283	.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	.22683	.47021	.12671	.4917	.03376	.51356	.94718	49
12	.44942	.2251	.47056	.12511	.49206	.03227	.51393	.94579	48
13	.44977	.22337	.47092	.1235	.49242	.03078	.5143	.9444	47
14	.45012	.22164	.47128	.1219	.49278	.02929	.51467	.94301	46
15	.45047	2.21992	.47163	2.1203	.49315	2.0278	.51503	1.94162	45
16	.45082	.21819	.47199	.11871	.49351	.02631	.5154	.94023	44
17	.45117	.21647	.47234	.11711	.49387	.02483	.51577	.93885	43
18	.45152	.21475	.4727	.11552	.49423	.02335	.51614	.93746	42
19	.45187	.21304	.47305	.11392	.49459	.02187	.51651	.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.9347	40
21	.45257	.20961	.47377	.11075	.49532	.01891	.51724	.93332	39
22	.45292	.2079	.47412	.10916	.49568	.01743	.51761	.93195	38
23	.45327	.20619	.47448	.10758	.49604	.01596	.51798	.93057	37
24	.45362	.20449	.47483	.106	.4964	.01449	.51835	.9292	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	.20108	.47555	.10284	.49713	.01155	.51909	.92645	34
27	.45467	.19938	.4759	.10126	.49749	.01008	.51946	.92508	33
28	.45502	.19769	.47626	.09969	.49786	.00862	.51983	.92371	32
29	.45537	.19599	.47662	.09811	.49822	.00715	.5202	.92235	31
30	.45573	2.1943	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	.19261	.47733	.09498	.49894	.00423	.52094	.91962	29
32	.45643	.19092	.47769	.09341	.49931	.00277	.52131	.91826	28
33	.45678	.18923	.47805	.09184	.49967	.00131	.52168	.9169	27
34	.45713	.18755	.4784	.09028	.50004	1.99986	.52205	.91554	26
35	.45748	2.18587	.47876	2.08872	.5004	1.99841	.52242	1.91418	25
36	.45784	.18419	.47912	.08716	.50076	.99695	.52279	.91282	24
37	.45819	.18251	.47948	.0856	.50113	.9955	.52316	.91147	23
38	.45854	.18084	.47984	.08405	.50149	.99406	.52353	.91012	22
39	.45889	.17916	.48019	.0825	50185	.99261	.5239	.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.4596	.17582	.48091	.07939	.50258	.98972	.52464	.90607	19
42	.45995	.17416	.48127	.07785	.50295	.98828	.52501	.90472	18
43	.4603	.17249	.48163	.0763	.50331	.98684	.52538	.90337	17
44	.46065	.17083	.48198	.07476	.50368	.9854	.52575	.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	.16751	.4827	.07167	.50441	.98253	.5265	.89935	14
47	.46171	.16585	.48306	.07014	.50477	.9811	.52687	.89801	13
48	.46206	.1642	.48342	.0686	.50514	.97966	.52724	.89667	12
49	.46242	.16255	.48378	.06706	.5055	.97823	.52761	.89533	11
50	.46277	2.1609	.48414	2.06553	.50587	1.9768	.52798	1.894	10
51	.46312	.15925	.4845	.064	.50623	.97538	.52836	.89266	9
52	.46348	.1576	.48486	.06247	.5066	.97395	.52873	.89133	8
53	.46383	.15596	.48521	.06094	.50696	.97253	.5291	.89	7
54	.46418	.15432	.48557	.05942	.50733	.97111	.52947	.88867	6
55	.46454	2.15268	.48593	2.0579	.50769	1.96969	.52984	1.88734	5
56	.46489	.15104	.48629	.05637	.50806	.96827	.53022	.88602	4
57	.46525	.1494	.48665	.05485	.50843	.96685	.53059	.88469	3
58	.4656	.14777	.48701	.05333	.50879	.96544	.53096	.88337	2
59	.46595	.14614	.48737	.05182	.50916	.96402	.53134	.88205	1
60	.46631	2.14451	.48773	2.0503	.50953	1.96261	.53171	1.88073	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	65°		64°		63°		62°		

′	28°		29°		30°		31°		
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	.87941	.55469	.80281	.57774	.73089	.60126	.66318	59
2	.53246	.87809	.55507	.80158	.57813	.72973	.60165	.66209	58
3	.53283	.87677	.55545	.80034	.57851	.72857	.60205	.66099	57
4	.5332	.87546	.55583	.79911	.5789	.72741	.60245	.6599	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	.87283	.55659	.79665	.57968	.72509	60324	.65772	54
7	.53432	.87152	.55697	.79542	.58007	.72393	.60364	.65663	53
8	.5347	.87021	.55736	.79419	.58046	.72278	.60403	.65554	52
9	.53507	.86891	.55774	.79296	.58085	.72163	.60443	.65445	51
10	.53545	1.8676	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	.8663	.5585	.79051	.58162	.71932	.60522	.65228	49
12	.5362	.86499	55888	.78929	.58201	.71817	.60562	.6512	48
13	.53657	.86369	.55926	.78807	.5824	.71702	.60602	.65011	47
14	.53694	.86239	.55964	.78685	.58279	.71588	.60642	.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	.85979	56041	.78441	.58357	.71358	.60721	.64687	44
17	.53807	.8585	.56079	.78319	.58396	.71244	.60761	.64579	43
18	.53844	.8572	56117	.78198	.58435	.71129	.60801	.64471	42
19	.53882	.85591	.56156	.78077	.58474	.71015	.60841	.64363	41
20	.5392	1.85462	56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	.85333	.56232	.77834	.58552	.70787	.60921	.64148	39
22	.53995	.85204	.5627	.77713	.58591	.70673	.6096	.64041	38
23	.54032	.85075	.56309	.77592	.58631	.7056	.61	.63934	37
24	.5407	.84946	.56347	.77471	.5867	.70446	.6104	.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.6108	1.63719	35
26	.54145	.84689	.56424	.7723	.58748	.70219	.6112	.63612	34
27	.54183	.84561	.56462	7711	.58787	.70106	.6116	.63505	33
28	.5422	.84433	565	.7699	.58826	.69992	.612	.63398	32
29	54258	.84305	.56539	.76869	.58865	.69879	.6124	.63292	31
30	.54296	1.84177	.56577	1.76749	.58904	1.69766	.6128	1.63185	30
31	.54333	.84049	.56616	.7663	.58944	.69653	6132	.63079	29
32	.54371	.83922	56654	7651	.58983	.69541	.6136	.62972	28
33	.54409	.83794	.56693	.7639	.59022	.69428	.614	.62866	27
34	.54446	.83667	.56731	.76271	.59061	.69316	.6144	6276	26
35	.54484	1.8354	.56769	1.76151	.59101	1.69203	6148	1.62654	25
36	.54522	.83413	.56808	.76032	.5914	.69091	6152	.62548	24
37	.5456	.83286	.56846	.75913	.59179	.68979	.61561	.62442	23
38	.54597	.83159	.56885	.75794	.59218	.68866	.61601	62336	22
39	.54635	.83033	.56923	75675	.59258	.68754	.61641	6223	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	.8278	.57	.75437	.59336	.68531	.61721	.62019	19
42	.54748	.82654	.57039	.75319	59376	.68419	.61761	.61914	18
43	.54786	.82528	.57078	752	59415	.68308	61801	.61808	17
44	.54824	.82402	.57116	75082	.59454	.68196	61842	.61703	16
45	.54862	1.82276	.57155	1.74964	59494	1.68085	61882	1.61598	15
46	.549	.8215	.57193	.74846	59533	.67974	61922	.61493	14
47	.54938	.82025	.57232	.74728	.59573	.67863	61962	.61388	13
48	.54975	.81899	57271	.7461	.59612	.67752	62003	.61283	12
49	.55013	.81774	.57309	.74492	.59651	.67641	.62043	.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.6753	.62083	1.61074	10
51	.55089	.81524	.57386	.74257	.5973	.67419	.62124	.6097	9
52	.55127	.81399	.57425	.7414	.5977	.67309	62164	.60865	8
53	.55165	.81274	.57464	.74022	.59809	.67198	.62204	.60761	7
54	.55203	.8115	.57503	.73905	.59849	.67088	.62245	.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	62285	1.60553	5
56	.55279	.80901	.5758	.73671	.59928	.66867	.62325	.60449	4
57	.55317	.80777	.57619	.73555	.59967	.66757	.62366	.60345	3
58	.55355	.80653	.57657	.73438	.60007	.66647	.62406	.60241	2
59	.55393	.80529	.57696	.73321	.60046	.66538	.62446	.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	61°		60°		59°		58°		

′	32°		33°		34°		35°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.624 87	1.600 33	.649 41	1.539 86	.674 51	1.482 56	.700 21	1.428 15	60
1	.625 27	.599 3	.649 82	.538 88	.674 93	.481 63	.700 64	.427 26	59
2	.625 68	.598 26	.650 23	.537 91	.675 36	.480 7	.701 07	.426 38	58
3	.626 08	.597 23	.650 65	.536 93	.675 78	.479 77	.701 51	.425 5	57
4	.626 49	.596 2	.651 06	.535 95	.676 2	.478 85	.701 94	.424 62	56
5	.626 89	1.595 17	.651 48	1.534 97	.676 63	1.477 92	.702 38	1.423 74	55
6	.627 3	.594 14	.651 89	.534	.677 05	.476 99	.702 81	.422 86	54
7	.627 7	.593 11	.652 31	.533 02	.677 48	.476 07	703 25	.421 98	53
8	.628 11	.592 08	.652 72	.532 05	.677 9	.475 14	.703 68	.421 1	52
9	.628 52	.591 05	.653 14	.531 07	.678 32	.474 22	.704 12	.420 22	51
10	.628 92	1.590 02	.653 55	1.530 1	.678 75	1.473 3	.704 55	1.419 34	50
11	.629 33	.589	.653 97	.529 13	.679 17	.472 38	.704 99	.418 47	49
12	.629 73	.587 97	.654 38	.528 16	.679 6	.471 46	.705 42	.417 59	48
13	.630 14	.586 95	.654 8	.527 19	.680 02	.470 53	.705 86	.416 72	47
14	.630 55	.585 93	.655 21	526 22	.680 45	.469 62	.706 29	.415 84	46
15	.630 95	1.584 9	.655 63	1.525 25	.680 88	1.468 7	.706 73	1.414 97	45
16	.631 36	.583 88	.656 04	.524 29	.681 3	.467 78	.707 17	.414 09	44
17	.631 77	.582 86	.656 46	.523 32	.681 73	.466 86	.707 6	.413 22	43
18	.632 17	.581 84	.656 88	.522 35	.682 15	.465 95	.708 04	.412 35	42
19	.632 58	.580 83	.657 29	.521 39	.682 58	.465 03	.708 48	.411 48	41
20	.632 99	1.579 81	.657 71	1.520 43	.683 01	1.464 11	.708 91	1.410 61	40
21	.633 4	.578 79	.658 13	.519 46	.683 43	.463 2	.709 35	.409 74	39
22	.633 8	.577 78	.658 54	.518 5	.683 86	.462 29	.709 79	.408 87	38
23	634 21	.576 76	.658 96	.517 54	684 29	.461 37	.710 23	.408	37
24	.634 62	.575 75	.659 38	.516 58	.684 71	.460 46	.710 66	.407 14	36
25	635 03	1.574 74	.659 8	1.515 62	.685 14	1.459 55	.711 1	1.406 27	35
26	.635 44	.573 72	.660 21	.514 66	.685 57	.458 64	.711 54	.405 4	34
27	.635 84	.572 71	.660 63	.513 7	.686	457 73	.711 98	.404 54	33
28	.636 25	.571 7	.661 05	.512 75	.686 42	.456 82	.712 42	.403 67	32
29	.636 66	.570 69	.661 47	.511 79	.686 85	.455 92	.712 85	.402 81	31
30	.637 07	1.569 69	.661 89	1.510 84	.687 28	1.455 01	.713 29	1.401 95	30
31	.637 48	.568 68	.662 3	.509 88	.687 71	.454 1	.713 73	.401 09	29
32	.637 89	.567 67	.662 72	.508 93	.688 14	.453 2	.714 17	.400 22	28
33	.638 3	.566 67	.663 14	.507 97	.688 57	.452 29	.714 61	.399 36	27
34	.638 71	.565 66	.663 56	.507 02	.689	.451 39	.715 05	.398 5	26
35	.639 12	1.564 66	.663 98	1.506 07	.689 42	1.450 49	.715 49	1.397 64	25
36	.639 53	.563 66	.664 4	.505 12	.689 85	.449 58	.715 93	396 79	24
37	.639 94	.562 65	.664 82	.504 17	.690 28	448 68	.716 37	.395 93	23
38	.640 35	.561 65	.665 24	.503 22	.690 71	.447 78	716 81	.395 07	22
39	640 76	.560 65	.665 66	.502 28	.691 14	.446 88	.717 25	394 21	21
40	641 17	1.559 66	.666 08	1.501 33	691 57	1.445 98	.717 69	1.393 36	20
41	.641 58	.558 66	.666 5	.500 38	692	.445 08	.718 13	.392 5	19
42	.641 99	557 66	.666 92	.499 44	692 43	.444 18	.718 57	.391 65	18
43	.642 4	.556 66	.667 34	.498 49	.692 86	.443 29	.719 01	.390 79	17
44	.642 81	.555 67	.667 76	.497 55	.693 29	.442 39	.719 46	.389 94	16
45	.643 22	1.554 67	.668 18	1.496 61	.693 72	1.441 49	.719 9	1.389 09	15
46	.643 63	.553 68	.668 6	.495 66	.694 16	.440 6	.720 34	.388 24	14
47	.644 04	.552 69	.669 02	.494 72	694 59	.439 7	.720 78	.387 38	13
48	.644 46	.551 7	.669 44	.493 78	.695 02	.438 81	.721 22	.386 53	12
49	.644 87	.550 71	.669 86	.492 84	.695 45	.437 92	.721 66	.385 68	11
50	.645 28	1.549 72	.670 28	1.491 9	.695 88	1.437 03	.722 11	1.384 84	10
51	.645 69	.548 73	.670 71	.490 97	.696 31	.430 14	.722 55	383 99	9
52	.646 1	.547 74	.671 13	.490 03	.696 75	.435 25	.722 99	.383 14	8
53	646 52	.546 75	.671 55	.489 09	697 18	.434 36	.723 44	.382 29	7
54	.646 93	545 76	.671 97	.488 16	.697 61	.433 47	.723 88	.381 45	6
55	.647 34	1.544 78	.672 39	1.487 22	.698 04	1.432 58	.724 32	1.380 6	5
56	.647 75	.543 79	.672 82	.486 29	.698 47	.431 69	.724 77	.379 76	4
57	.648 17	.542 81	.673 24	.485 36	.698 91	.430 8	.725 21	.378 91	3
58	.648 58	.541 83	.673 66	.484 42	.699 34	.429 92	.725 65	.378 07	2
59	.648 99	.540 85	.674 09	.483 49	.699 77	.429 03	.726 1	.377 22	1
60	.649 41	1.539 86	.674 51	1.482 56	.700 21	1.428 15	.726 54	1.376 38	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	57°		56°		55°		54°		

′	36°		37°		38°		39°		′
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.2349	60
1	.72699	.37554	.75401	.32624	.78175	.27917	.81027	.23416	59
2	.72743	.3747	.75447	.32544	.78222	.27841	.81075	.23343	58
3	.72788	.37386	.75492	.32464	.78269	.27764	.81123	.2327	57
4	.72832	.37302	.75538	.32384	.78316	.27688	.81171	.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.8122	1.23123	55
6	.72921	.37134	.75629	.32224	.7841	.27535	.81268	.2305	54
7	.72966	.3705	.75675	.32144	.78457	.27458	.81316	.22977	53
8	.7301	.36967	.75721	.32064	.78504	.27382	.81364	.22904	52
9	.73055	.36883	.75767	.31984	.78551	.27306	.81413	.22831	51
10	.731	1.368	.75812	1.31904	.78598	1.2723	.81461	1.22758	50
11	.73144	.36716	.75858	.31825	.78645	.27153	.8151	.22685	49
12	.73189	.36633	.75904	.31745	.78692	.27077	.81558	.22612	48
13	.73234	.36549	.7595	.31666	.78739	.27001	.81606	.22539	47
14	.73278	1.36466	.75996	.31586	.78786	.26925	.81655	.22467	46
15	.73323	.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	.363	.76088	.31427	.78881	.26774	.81752	.22321	44
17	.73413	.36217	.76134	.31348	.78928	.26698	.818	.22249	43
18	.73457	.36133	.7618	.31269	.78975	.26622	.81849	.22176	42
19	.73502	.36051	.76226	.3119	.79022	.26546	.81898	.22104	41
20	.73547	1.35968	.76272	1.3111	.7907	1.26471	.81946	1.22031	40
21	.73592	.35885	.76318	.31031	.79117	.26395	.81995	.21959	39
22	.73637	.35802	.76364	.30952	.79164	.26319	.82044	.21886	38
23	.73681	.35719	.7641	.30873	.79212	.26244	.82092	.21814	37
24	.73726	.35637	.76456	.30795	.79259	.26169	.82141	.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.8219	1.2167	35
26	.73816	.35472	.76548	.30637	.79354	.26018	.82238	.21598	34
27	.73861	.35389	.76594	.30558	.79401	.25943	.82287	.21526	33
28	.73906	.35307	.7664	.3048	.79449	.25867	.82336	.21454	32
29	.73951	.35224	.76686	.30401	.79496	.25792	.82385	.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.2131	30
31	74041	.3506	.76779	.30244	.79591	.25642	.82483	.21238	29
32	.74086	.34978	.76825	.30166	.79639	.25567	.82531	.21166	28
33	.74131	.34896	.76871	.30087	.79686	.25492	.8258	.21094	27
34	.74176	.34814	.76918	.30009	.79734	.25417	.82629	.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	.3465	.7701	.29853	.79829	.25268	.82727	.20879	24
37	.74312	.34568	.77057	.29775	.79877	.25193	.82776	.20808	23
38	.74357	.34487	.77103	.29696	.79924	.25118	.82825	.20736	22
39	.74402	.34405	.77149	.29618	.79972	.25044	.82874	.20665	21
40	.74447	1.34323	.77196	1.29541	.8002	1.24969	.82923	1.20593	20
41	.74492	.34242	.77242	.29463	.80067	.24895	.82972	.20522	19
42	.74538	.3416	.77289	.29385	.80115	.2482	.83022	.20451	18
43	.74583	.34079	.77335	.29307	.80163	.24746	.83071	.20379	17
44	.74628	.33998	.77382	.29229	.80211	.24672	.8312	.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	.33835	.77475	.29074	.80306	.24523	.83218	.20166	14
47	.74764	.33754	.77521	.28997	.80354	.24449	.83268	.20095	13
48	.7481	.33673	.77568	.28919	.80402	.24375	.83317	.20024	12
49	.74855	.33592	.77615	.28842	.8045	.24301	.83366	.19953	11
50	.749	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	.3343	.77708	.28687	.80546	.24153	.83465	.19811	9
52	.74991	.33349	.77754	.2861	.80594	.24079	.83514	.1974	8
53	.75037	.33268	.77801	.28533	.80642	.24005	.83564	.19669	7
54	.75082	.33187	.77848	.28456	.8069	.23931	.83613	.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	.33026	.77941	.28302	.80786	.23784	.83712	.19457	4
57	.75219	.32946	.77988	.28225	.80834	.2371	.83761	.19387	3
58	.75264	.32865	.78035	.28148	.80882	.23637	.83811	.19316	2
59	.7531	.32785	.78082	.28071	.8093	.23563	.8386	.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.2349	.8391	1.19175	0
′	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	′
	53°		52°		51°		50°		

	40°		41°		42°		43°		
	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	
0	.8391	1.19175	.86929	1.15037	.9004	1.11061	.93252	1.07237	60
1	.8396	19105	.8698	.14969	.90093	.10996	.93306	.07174	59
2	.84009	.19035	.87031	.14902	.90146	.10931	.9336	.07112	58
3	.84059	.18964	.87082	.14834	.90199	.10867	.93415	.07049	57
4	.84108	.18894	.87133	.14767	.90251	.10802	.93469	.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	.18754	.87236	.14632	.90357	.10672	.93578	.06862	54
7	.84258	18684	.87287	.14565	.9041	.10607	.93633	.068	53
8	.84307	.18614	.87338	.14498	.90463	.10543	.93688	06738	52
9	.84357	.18544	.87389	.1443	.90516	.10478	.93742	.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	.18404	.87492	.14296	.90621	.10349	.93852	.06551	49
12	.84507	.18334	.87543	.14229	.90674	.10285	.93906	.06489	48
13	.84556	.18264	.87595	.14162	.90727	.1022	.93961	.06427	47
14	.84606	.18194	.87646	.14095	.90781	.10156	.94016	.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	94071	1.06303	45
16	.84706	.18055	.87749	.13961	.90887	.10027	.94125	.06241	44
17	.84756	.17986	.87801	.13894	.9094	.09963	.9418	.06179	43
18	.84806	.17916	.87852	.13828	.90993	.09899	.94235	.06117	42
19	.84856	.17846	.87904	.13761	.91046	.09834	.9429	.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.0977	.94345	1.05994	40
21	.84956	.17708	.88007	.13627	.91153	.09706	.944	.05932	39
22	.85006	.17638	.88059	.13561	.91206	.09642	.94455	.0587	38
23	.85057	.17569	.8811	.13494	.91259	.09578	.9451	.05809	37
24	.85107	.175	.88162	.13428	.91313	.09514	.94565	.05747	36
25	.85157	1.1743	.88214	1.13361	.91366	1.0945	.9462	1.05685	35
26	.85207	.17361	.88265	.13295	.91419	.09386	.94676	.05624	34
27	.85257	.17292	.88317	.13228	.91473	.09322	.94731	.05562	33
28	.85307	.17223	.88369	.13162	.91526	.09258	.94786	.05501	32
29	.85358	.17154	.88421	.13096	.9158	.09195	.94841	.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	.17016	.88524	12963	.91687	.09067	.94952	.05317	29
32	.85509	.16947	.88576	.12897	.9174	.09003	.95007	.05255	28
33	.85559	.16878	.88628	.12831	.91794	.0894	.95062	.05194	27
34	85609	.16809	.8868	.12765	.91847	.08876	.95118	.05133	26
35	.8566	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.8571	.16672	.88784	.12633	.91955	.08749	.95229	.0501	24
37	.85761	.16603	.88836	.12567	.92008	.08686	.95284	.04949	23
38	.85811	.16535	.88888	.12501	.92062	.08622	.9534	.04888	22
39	.85862	.16466	.8894	.12435	.92116	.08559	.95395	.04827	21
40	.85912	1.16398	.88992	1.12369	.9217	1.08496	.95451	1.04766	20
41	.85963	.16329	.89045	.12303	.92223	.08432	.95506	.04705	19
42	.86014	.16261	.89097	.12238	.92277	.08369	.95562	.04644	18
43	.86064	.16192	.89149	.12172	.92331	.08306	.95618	.04583	17
44	.86115	.16124	.89201	.12106	.92385	.08243	.95673	.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	.15987	.89306	.11975	.92493	.08116	.95785	.04401	14
47	.86267	.15919	.89358	.11909	.92547	.08053	.95841	.0434	13
48	.86318	15851	.8941	.11844	.92601	.0799	.95897	.04279	12
49	.86368	.15783	.89463	.11778	.92655	.07927	.95952	.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.8647	.15647	.89567	.11648	.92763	.07801	.96064	.04097	9
52	.86521	.15579	.8962	.11582	.92817	.07738	.9612	.04036	8
53	.86572	.15511	.89672	.11517	.92872	.07676	.96176	.03976	7
54	.86623	.15443	.89725	.11452	.92926	.07613	.96232	.03915	6
55	.86674	1.15375	.89777	1.11387	.9298	1.0755	96288	1.03855	5
56	.86725	.15308	.8983	.11321	.93034	.07487	.96344	.03794	4
57	.86776	.1524	.89883	.11256	.93088	.07425	.964	.03734	3
58	.86827	.15172	.89935	.11191	.93143	.07362	.96457	.03674	2
59	.86878	.15104	.89988	.11126	.93197	.07299	.96513	.03613	1
60	.86929	1.15037	.9004	1.11061	.93252	1.07237	.96569	1.03553	0
	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	Co-tang.	Tang.	
	49°		48°		47°		46°		

	44°				44°				44°		
′	Tang.	Co-tang.	′	′	Tang.	Co-tang.	′	′	Tang.	Co-tang.	′
0	.965 69	1.035 53	60	21	.977 56	1.022 95	39	41	.989 01	1.011 12	19
1	.966 25	1.034 93	59	22	.978 13	1.022 36	38	42	.989 58	1.010 53	18
2	.966 81	1.034 33	58	23	.978 7	1.021 76	37	43	.990 16	1.009 94	17
3	.967 38	1.033 72	57	24	.979 27	1.021 17	36	44	.990 73	1.009 35	16
4	.967 94	1.033 12	56	25	.979 84	1.020 57	35	45	.991 31	1.008 76	15
5	.968 5	1.032 52	55	26	.980 41	1.019 98	34	46	.991 89	1.008 18	14
6	.969 07	1 031 92	54	27	.980 98	1.019 39	33	47	.992 .7	1.007 59	13
7	.969 63	1.031 32	53	28	.981 55	1.018 79	32	48	.993 04	1.007 01	12
8	.970 2	1.030 72	52	29	.982 13	1.018 2	31	49	.993 62	1 006 42	11
9	.970 76	1.030 12	51	30	.982 7	1.017 61	30	50	.994 2	1.005 83	10
10	.971 33	1.029 52	50	31	.983 27	1.017 02	29	51	.994 78	1.005 25	9
11	971 89	1.028 92	49	32	.983 84	1.016 42	28	52	.995 36	1.004 67	8
12	.972 46	1.028 32	48	33	.984 41	1.015 83	27	53	.995 94	1 004 08	7
13	973 02	1.027 72	47	34	.984 99	1.015 24	26	54	.996 52	1.003 5	6
14	973 59	1.027 13	46	35	.985 56	1.014 65	25	55	.997 1	1.002 91	5
15	.974 16	1.026 53	45	36	.986 13	1.014 06	24	56	.997 68	1.002 33	4
16	.974 72	1.025 93	44	37	.986 71	1.013 47	23	57	.998 26	1.001 75	3
17	.975 29	1.025 33	43	38	.987 28	1.012 88	22	58	.998 84	1.001 16	2
18	.975 86	1.024 74	42	39	.987 86	1.012 29	21	59	.999 42	1.000 58	1
19	.976 43	1.024 14	41	40	.988 43	1.011 7	20	60	1	1	0
20	.977	1.023 55	40								
′	Co-tang.	Tang.	′	′	Co-tang.	Tang.	′	′	Co-tang.	Tang.	′
	45°				45°				45°		

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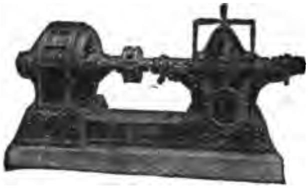

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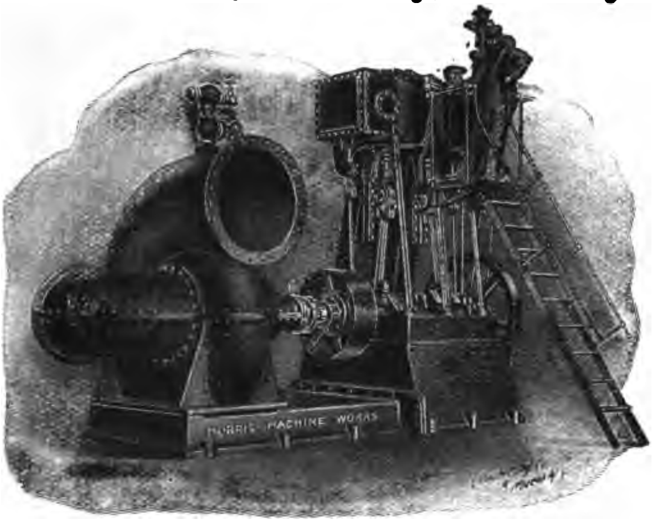

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